Chapter 8: Solving Quadratic Equations

Section 8.1 Revising Quadratic Factorisation

Remember Quadratic Factorisation

- The expression inside each set of brackets will start with an *x*, and the signs in the quadratic expression show which signs to put after the *x*s.
- When the *second* sign in the expression is a ***plus***, the signs in both sets of brackets are the *same* as the *first sign*.

$x^2 + ax + b = (x + ?)(x + ?)$ Since everything is positive.

$x^2 – ax + b = (x - ?)(x - ?)$ Since negative × negative = positive

- Next, look at the *last number*, b, in the expression. When ***multiplied*** together, the two numbers in the brackets must give b.
- Finally, look at the coefficient of *x*, which is *a*. The sum of the two numbers in the brackets will give *a*.

Factorise $\mathbf{x^2 + 5x + 6}$
$(x + 2)(x + 3)$

Factorise $\mathbf{x^2 – 7x + 10}$
$(x – 5)(x – 2)$

Factorisation of $x^2 – x – 6$ gives $(x – 3)(x + 2)$

- When the *second* sign is a ***minus***, the signs in the brackets are *different*.

$x^2 + ax – b = (x + ?)(x - ?)$ Since positive × negative = negative

$x^2 – ax – b = (x + ?)(x - ?)$

The ***larger*** factor will have the minus sign before it.

- Next, look at the *last number*, b, in the expression. When ***multiplied*** together, the two numbers in the brackets must give b.

Finally, look at the coefficient of *x*, which is *a*. The sum of the two numbers in the brackets will give *a*.

Factorise $\mathbf{x^2 + 3x - 18}$

$(x + 6)(x - 3)$

Difference of Two Squares

Factorise $\mathbf{x^2 - 36}$

Recognise the difference of two squares x^2 and 6^2

So it factorises to $(x + 6)(x - 6)$

To check your answer, expand the brackets once again.

Section 8.2 Solving Quadratic Equations (Type 1)

To solve a quadratic equation, the first step is to write it in the form:

$$ax^2 + bx + c = 0$$

where $a \neq 0$, and b and c are integers.

Then factorise the equation as we have revised in the previous section.

Solving Quadratic Equations by factorization depends on having 0 on one side of the equation and so sometimes it is necessary to rearrange the equation into the form $ax^2 + bx + c = 0$ first.

Any quadratic equation gives 2 values as answers.

Example 1

Solve $x(x + 5) = 0$

If $x(x + 5) = 0$

$\therefore x = 0$ and $x + 5 = 0$

$x = -5$

Example 2

Solve: $3x(x - 3) = 0$

If $3x(x - 3) = 0$

$\therefore$ $3x = 0$ and $x - 3 = 0$

$x = 0$ $x = 3$

Example 3

Solve: $(x - 3)(5x + 4) = 0$

If $(x - 3)(5x + 4) = 0$

$\therefore$ $x - 3 = 0$ and $5x + 4 = 0$

$x = 3$ $5x = -4$

$x = -\frac{4}{5}$

Consolidation

Solve the following:

1. $x(x + 1) = 0$

2. $x(x - 7) = 0$

3. $2x(2x - 1) = 0$

4. $5x(3 - 5x) = 0$

5. $3x(2 + 3x) = 0$

6. $(x - 5)(x - 4) = 0$

7. $(4x - 3)(3x - 1) = 0$

Support Exercise Pg 488 Exercise 30A Nos 7 – 11

Handout

Section 8.3 Solving Quadratic Equations (Type 2)

Here you are given two terms, one in x^2 and the other in x. Start by taking x as the common factor. Our result has to be similar to the ones in the previous section.

Example 1

Solve: $5x^2 - 4x = 0$

If $5x^2 - 4x = 0$

$x(5x - 4) = 0$

$\therefore$ $x = 0$ and $5x - 4 = 0$

$x = \frac{4}{5}$

Example 2

Solve: $6x^2 - 9x = 0$

If $6x^2 - 9x = 0$

$3x(2x - 3) = 0$

$\therefore$ $3x = 0$ and $2x - 3 = 0$

$x = 0$ $2x = 3$

$x = 3/2$

$x = 1\frac{1}{2}$

Consolidation

Solve the following:

1. $3x^2 - 2x = 0$

2. $x^2 + 3x = 0$

3. $4x^2 + x = 0$

4. $2x^2 - 6x = 0$

5. $9x^2 + 3x = 0$

Support Exercise Pg 488 Exercise 30A Nos 12 – 16

Handout

Section 8.4 Solving Quadratic Equations (Type 3)

Square Root Method

Example 1

Solve: $x^2 = 25$

If $x^2 = 25$

$x = \pm\sqrt{25}$

$\therefore$ $x = \pm 5$

NOTE: Remember that when you find the square root of a number, the result may be *positive* or *negative*.

Plus or Minus Sign

$\pm$ is a special symbol that means 'plus or minus'

so instead of writing $w = \sqrt{a}$ or $w = -\sqrt{a}$

we can write $w = \pm\sqrt{a}$

In a Nutshell

When we have: $r^2 = x$

then: $r = \pm\sqrt{x}$

Example 2

Solve: $2x^2 = 72$

If $2x^2 = 72$

$x^2 = 72 \div 2$

$x^2 = 36$

$x = \sqrt{36}$

∴ $x = \pm 6$

Example 3

Solve: $2x^2 - 50 = 0$

If $2x^2 - 50 = 0$

$2x^2 = 50$

$x^2 = 50 \div 2$

$x^2 = 25$

$x = \sqrt{25}$

∴ $x = \pm 5$

Example 4

Solve: $64x^2 - 1 = 0$

If $64x^2 - 1 = 0$

$64x^2 = 1$

$$x^2 = \frac{1}{64}$$

$$x = \sqrt{\frac{1}{64}}$$

∴ $$x = \pm\frac{1}{8}$$

Example 5

Solve: $\frac{4}{x} - x = 0$

If $\frac{4}{x} - \frac{x}{1} = \frac{0}{1}$

The LCM of x, 1 and 1 is x

$\therefore$ $\frac{4 - x^2 = 0}{x}$

$4 - x^2 = 0$

$4 = x^2$

$\sqrt{4} = x$

$\therefore$ $x = \pm 2$

Example 6

NOTE: In this type you are given a bracket to the power of 2 equals to a number. To solve this equation you have to find the square root of each side.

Solve: $(2x - 1)^2 = 25$

If $(2x - 1)^2 = 25$

$2x - 1 = \sqrt{25}$

$2x - 1 = \pm 5$

$\therefore$ $2x - 1 = 5$ and $2x - 1 = -5$

$2x = 5 + 1$ $2x = -5 + 1$

$2x = 6$ $2x = -4$

$x = 3$ $x = -2$

Consolidation

Solve the following:

1. $x^2 = 16$

2. $2x^2 = 8$

3. $2x^2 - 18 = 0$

4. $9x^2 - 1 = 0$

5. $x - \frac{9}{x} = 0$

6. $(x + 1)^2 = 9$

7. $(2x + 3)^2 = 50$

Support Exercise Pg 488 Exercise 30A Nos 1 – 5

Handout

Section 8.5 Solving Quadratic Equations (Type 4)

Example 1

Solve the equation $x^2 - 9x + 20 = 0$

Solution

1) First, factorise the quadratic equation $x^2 - 9x + 20 = 0$

   Find two numbers which add up to 9 and multiply to give 20. These numbers are 4 and 5.

   $(x - 4)(x - 5) = 0$

2) Now find the value x so that when these brackets are multiplied together the answer is 0.

   This means either

   $(x - 4) = 0$ or $(x - 5) = 0$

   So $x = 4$ or $x = 5$.

3) You can check these answers by substitutuing 4 and 5 in to the equation:

   $x^2 - 9x + 20$

   Substituting 4 gives:
   $4^2 - 9 \times 4 + 20 = 16 - 36 + 20 = 0$

   Substituting 5 gives:
   $5^2 - 9 \times 5 + 20 = 25 - 45 + 20 = 0$

Remember these 3 simple steps and you will be able to solve quadratic equations.

Example 2

Solve $\mathbf{x^2 + 6x + 5 = 0}$

This factorises into $(x + 5)(x + 1) = 0$

The only way this expression can ever equal 0 is if the value of one of the brackets is 0.

Hence either

$x + 5 = 0$ or $x + 1 = 0$

$x = -5$ $x = -1$

So the solution is $x = -5$ or $x = -1$.

There are two possible values of x.

Example 3

Solve $\mathbf{x^2 + 3x - 10 = 0}$

This factorises into $(x + 5)(x - 2) = 0$

Hence

$x + 5 = 0$ or $x - 2 = 0$

$x = -5$ $x = 2$

Example 4

Solve $\mathbf{x^2 - 6x + 9 = 0}$

This factorises into $(x - 3)(x - 3) = 0$

This equation has repeated roots.

That is: $(x - 3)^2 = 0$

Hence, there is only one solution, $x = 3$

Consolidation

Solve the following equation:

1. $x^2 + 5x + 4 = 0$

2. $x^2 - 8x + 15 = 0$

3. $t^2 + 4t - 12 = 0$

4. $x^2 + 4x + 4 = 0$

5. $x^2 + 10x = - 24$

Support Exercise Pg 488 Exercise 30A Nos 17 – 25

Handout

Section 8.6 Solving problems using quadratic equations

Example 1

A woman is x years old. Her husband is three years younger. The product of their ages is 550.

a) Set up a quadratic equation to represent this situation.

Woman : x

Husband : x – 3

Product means Multiplication

x(x – 3) = 550

b) How old is the woman?

$x(x - 3) = 550$

$x^2 - 3x - 550 = 0$

$(x - 25)(x + 22) = 0$

$x - 25 = 0$ and $x + 22 = 0$

$x = 25$ $x = -22$

The woman is 25 years old.

Example 2

A rectangular field is 40m longer than it is wide.

The area is 48 000 square metres.

The farmer wants to place a fence all around the field.

How long will the fence be?

Width = x m

Length = x + 40 m

Area = Length $\times$ Breadth

$x(x + 40) = 48\ 000$

$x^2 + 40x = 48\ 000$

$x^2 + 40x - 48\ 000 = 0$

$(x + 240)(x - 200) = 0$

$x + 240 = 0$ and $x - 200 = 0$

$x = -240$ $x = 200$

Let x = 200m

Width = 200m

Length = 200 + 40 = 240m

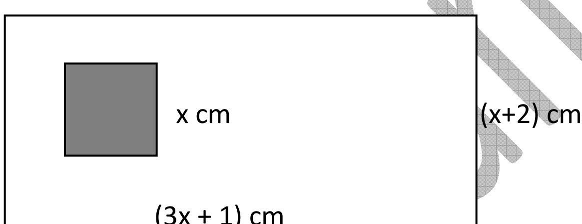
Perimeter = 2(200 + 240) = 880 m

Consolidation

1. A rectangular lawn is 2m longer than it is wide.

   The area of the lawn is $21m^2$. The gardener wants to edge the lawn with edging strips, which are sold in lengths of 1 ½ m. How many will she need to buy?

2. A square of side x cm is removed from a rectangular piece of cardboard measuring (3x+1) cm and (x+2)cm. If the area of a card remaining is $62cm^2$ form an equation in x and solve it to find the dimensions of the original card.



3. The sides of a right-angled triangle are x cm, (x+7) cm and (x+8) cm. Find them.

4. The sum of 2 numbers is 13 and the sum of their squares is 85. Find them.

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