Chapter 8: Algebra Part 2

Section 8.1 Algebraic Products

<u>Expanding</u> brackets means to ***<u>remove</u>*** the brackets.

How would we expand the following?

5 (x + 2)

The term which is outside the brackets must be multiplied with the WHOLE bracket.

(Multiply term by term)

5 (x + 2) = (5 × x) + (5 x 2)

= 5x + 10

Consolidation

1. 2(x+1) ____________________
2. 4(2 + 3b) ____________________
3. 3(b + 2f) ____________________

<u>Expanding with letters outside the bracket</u>

Expand: p(p + 4)

p (p + 4) = (p x p) + (p x 4)

= p^2 + 4p

<u>Remember:</u>

Positive (+) x Negative (-) = Negative (-)

Negative (-) x Negative (-) = Positive (+)

Positive (+) x Positive (+) = Positive (+)

Consolidation

1. r(2r – 6) ____________________
2. –s (s+5) ____________________
3. $2y(y^2 +3y + 6)$ ____________________

<u>Expanding two brackets at a go</u>

Expand: 6q – 2(r – 2q) = 6q - (2 x r) – (2 x (-2q))

= 6q – 2r + 4q *(remember Negative x Negative = Positive)*

= 10q – 2r *(Collect like terms)*

Consolidation

1. 3(y + 2z) + 5(2y + 3z)

2. 6n(n – 3) – 5(n + 2)

3. 3q – 2(q – 6)

4. t(t – 6) – 3(t – 3)

An expression such as $(3y + 2)(4y – 5)$ can be expanded to give a quadratic expression.

Multiplying out such pairs of brackets is usually called **quadratic expansion**.

The rule for expanding expressions such as $(t + 5)(3t – 4)$ is similar to that for expanding single brackets: multiply everything in one set of brackets by everything in the other set of brackets.

Example 1

In the expansion method, split the terms in the first set of brackets, make each of them multiply both terms in the second set of brackets, then simplify the outcome.

Expand $(x + 3)(x + 4)$

$(x + 3)(x + 4) = x(x + 4) + 3(x + 4)$

$= x^2 + 4x + 3x + 4$

$= \mathbf{x^2 + 7x + 12}$

Example 2

Expand $(t + 5)(t – 2)$

$(t + 5)(t – 2) = t(t – 2) + 5(t – 2)$

$= t^2 – 2t + 5t – 10$

$= \mathbf{t^2 + 3t – 10}$

Example 3

Expand $(k – 3)(k – 2)$

$(k – 3)(k – 2) = k(k – 2) – 3(k – 2)$

$= k^2 – 2k – 3k + 6$

$= \mathbf{k^2 – 5k + 6}$

Example 4

Expand $(4x - 1)(3x - 5)$

$(4x - 1)(3x - 5) = 4x(3x - 5) - 1(3x - 5)$

$= 12x^2 - 20x - 3x + 5$

$= \mathbf{12x^2 - 23x + 5}$

Example 5

Expand $(3x - 2)^2$

$(3x - 2)^2 = (3x - 2)(3x - 2)$

$= 3x(3x - 2) - 2(3x - 2)$

$= 9x^2 - 6x - 6x + 4$

$= \mathbf{9x^2 - 12x + 4}$

Consolidation

Expand the following:

1. $(w + 3)(w - 1)$

2. $(m + 5)(m + 1)$

3. $(a - 1)(a - 3)$

4. (x + 3)(x – 3)

5. (4r – 3)(2r – 1)

6. (1 – 3p)(3 + 2p)

7. $(t - 5)^2$

8. $(x + 6)^2 - 36$

Support Exercise Pg 107 Exercise 8A

Pg 110 Exercise 8C Nos 1 – 4

Section 8.2 Factorisation

Factorisation is the opposite of expansion. It puts an expression back into the brackets it may have come from.

In factorization you have to look for the **common factors** in every term of the expression.

Example 1

$6t + 9m = 3(2t + 3m)$

3 is a factor if 6 and 9

Example 2

$6my + 4py = 2y\ (3m + 2p)$

2 and y are in both terms.

Example 3

$5k^2 – 25k = 5k\ (k – 5)$

Example 4

$10a^2b – 15ab^2 = 5ab\ (2a – 3b)$

Consolidation

1. $4t^2 – 3t$ ______________________________
2. $3m^2 – 3mp$ ______________________________
3. $6ab + 9bc + 3bd$ ______________________________
4. $6mt^2 – 3mt + 9m^2t$ ______________________________
5. $8ab^2 + 2ab – 4a^2b$ ______________________________

Support Exercise Pg 108 Ex 8B Nos 1, 2

Section 8.3 Solving Linear Equations

Some equations can be solved mentally. To solve more complicated equations the **balance method** is used.

To keep the balance, whatever you do on the left-hand side you must also do to the right hand side of the equation.

It is easier to remember:

CHANGE side …. → CHANGE sign

Example 1

4x + 3 = 31

4x = 31 – 3

4x = 28

x = 28 ÷ 4

x = 7

Example 2

5(a + 3) = 18

5a + 15 = 18

5a = 18 – 15

a = $^{3}/_{5}$

Example 3

5(3y + 2) = 13y + 4

15y + 10 = 13y + 4

15y – 13y = 4 – 10

2y = -6

y = -6 ÷ 2

y = -3

Consolidation

1. 3x + 2 = 14

2. 4(2x – 4) = 8

3. 3x + 8 = 2x – 4

4. 2(3x + 4) = 4(2x – 3)

5. 5(2x + 7) = -15

Support Exercise Pg 144 Ex 10B 1 – 40

Section 8.4 Setting Up Equations

Equations are used to represent situations, so that you can solve real-life problems. Many real-life problems can be solved by setting them up as linear equations and then solving the equation.

Example 1

A man buys a daily news paper from Monday to Saturday for d cents. He buys a Sunday paper for 1.80 dollars. His weekly paper bill is 7.20 dollars.

What is the price of his daily paper?

6d + 180 = 720

6d = 720 – 180

6d = 540

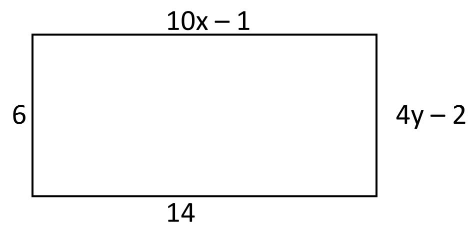
d = 540 ÷ 6

d = 90

Therefore the daily paper costs 90 cents.

Consolidation

1. The diagram shows a rectangle.



a) What is the value of x?

b) What is the value of y?

2. Marisa has two bags, each of which contains the same number of sweets. She eats 4 sweets. She then finds that she has 30 sweets left. How many sweets were there in each bag to start with?

3. Flooring costs $12.75 per square meter. The shop charges $35 for fitting. The final bill was $137. How many square meters of flooring were fitted?

4. Mario bought 8 garden chairs. When he got to the till he used a $10 voucher as part payment. His final bill was $56.

   a. Set this problem up as an equation, using c as the cost of one chair.

   ____

   b. Solve the equation to find the cost of one chair.

   ____

   ____

   ____

   ____

Support Exercise Pg 146 Ex 10B Nos 1 – 15

Section 8.5 Solving Fractional Terms

In algebra expressions such as (y + 5) ÷ 4 are usually written as $\frac{y+5}{4}$

<u>Example 1:</u>

Solve the equation $\frac{4}{q} = 8$

Step 1: Remove the denominator from the equation

Multiply <u>*both*</u> sides by q: $\frac{4}{\cancel{q}} \times \cancel{q} = 8 \times q$

4 = 8q

Step 2: Equate the unknown

Divide <u>*both*</u> sides by 8: $\frac{4}{8} = \frac{\cancel{8}q}{\cancel{8}}$

Simplify: $q = \frac{1}{2}$

We can reduce the steps and conduct some of them mentally. Have a look at the next example:

Example 2:

Solve the equation $\frac{5}{y} = 25$

Example 3:

Solve the equation: $\frac{3(q+5)}{2} = 6$

Example 4:

Solve the equation $\frac{16-x}{4}=1-x$

Example 5:

Solve the equation $\frac{2x-1}{2}-\frac{x-5}{3}=\frac{5}{4}$

Support Exercise Pg 150 Ex 10C Nos 1 – 5

Section 8.6 Solving with Square Root

NOTE: Remember that when you find the square root of a number, the result may be *positive* or *negative*.

Plus or Minus Sign

$\pm$ is a special symbol that means 'plus or minus'

so instead of writing $w = \sqrt{a}$ or $w = -\sqrt{a}$

we can write $w = \pm\sqrt{a}$

In a Nutshell

When we have: $r^2 = x$

then: $r = \pm\sqrt{x}$

Example 1

Solve: $2x^2 = 72$

If $2x^2 = 72$

$x^2 = 72 \div 2$

$x^2 = 36$

$x = \sqrt{36}$

$\therefore$ $x = \pm 6$

Example 2

Solve: $2x^2 - 50 = 0$

If $2x^2 - 50 = 0$

$2x^2 = 50$

$x^2 = 50 \div 2$

$x^2 = 25$

$x = \sqrt{25}$

$\therefore$ $x = \pm 5$

Example 3

NOTE: In this type you are given a bracket to the power of 2 equals to a number. To solve this equation you have to find the square root of each side.

Solve: $(2x - 1)^2 = 25$

If $(2x - 1)^2 = 25$

$2x - 1 = \sqrt{25}$

$2x - 1 = \pm 5$

$\therefore$ $2x - 1 = 5$ and $2x - 1 = -5$

$2x = 5 + 1$ $2x = -5 + 1$

$2x = 6$ $2x = -4$

$x = 3$ $x = -2$

Consolidation

1. $2x^2 = 8$

2. $2x^2 - 18 = 0$

3. $(2x + 3)^2 = 50$

4. $(x - 5)^2 - 100 = 0$

Support Exercise Pg 488 Exercise 30A Nos 1 – 5

Section 8.7 Solving Simultaneous Equations

Pair of simultaneous equations are two linear equations for which you have two unknowns and a solution for each is required.

Elimination Method

Step 1

Get the **coefficients** of one of the unknowns the same.

Step 2

Eliminate this unknown by **adding** or **subtracting** the two equations. (When the signs are the same you subtract; when the signs are different add up the equations)

Step 3

Solve the resulting equation with one unknown.

Step 4

Substitute the value found back into any one of the original equations.

Step 5

Solve the resulting equation.

Step 6

Check that the two values found satisfy the original equations.

Example 1

Solve the equations: ***6x + y = 15*** *and* ***4x + y = 11***

Label the equations so that the method can be clearly explained.

$6x + y = 15$ (1)

$4x + y = 11$ (2)

Step 1: Since the y-term in both equations has the same coefficient there is no need to balance term.

Step 2: Subtract one equation from the other. (Equation (1) minus equation (2) will give positive values.)

(1) – (2) $2x = 4$

Step 3: $x = 4 \div 2$

$x = 2$

Step 4: Substitute x = 2 into one of the original equations. (Usually the one with the smallest values is the easiest)

So substitute into: $4x + y = 11$

Which gives: $4(2) + y = 11$

Step 5: Solve this equation: $8 + y = 11$

$y = 11 - 8$

$y = 3$

Step 6: Test the solution in the original equations. So substitute x = 2 and y = 3 into 6x + y, which gives 12 + 3 = 15 and into 4x + y, which gives 8 + 3 = 11. These are correct, so you can confidently say that the solution is x = 2 and y = 3.

Example 2

Solve these equations. ***3x + 2y = 18*** *(1)*

2x – y = 5 *(2)*

Step 1: Multiply equation (2) by 2. There are other ways to balance the coefficients but this is the easiest and leads to less work later. With practice, you will get used to which will be the best way to balance the coefficients.

2 × (2) $4x - 2y = 10$ (3)

Label this equation as equation (3)

Be careful to multiply every term and not just the y-term. You could write:

$$2 \times (2x - y = 5) \rightarrow 4x - 2y = 10 \qquad (3)$$

Step 2: As the signs of the y-terms are opposite, add the equations.

(1) + (3) $7x = 28$

Be careful to add the correct equations. This is why labeling them is useful.

Step 3: Solve this equation: $x = 28 \div 7$

$x = 4$

Step 4: Substitute $x = 4$ into any equation, say $2x - y = 5 \rightarrow 8 - y = 5$

Step 5: Solve the equation: $8 - 5 = y$

$y = 3$

Step 6: Check: (1), $3 \times 4 + 2 \times 3 = 18$ and (2), $2 \times 4 - 3 = 5$, which are correct so the solution is $x = 4$ and $y = 3$.

Example 3

Solve these equations: $\boldsymbol{4x + 3y = 27}$ *(1)*

$\boldsymbol{5x - 2y = 5}$ *(2)*

Both equations have to be changed to obtain identical terms in either x or y.

However, you can see that if you make the y-coefficients the same, you will add the equations. This is always safer than subtraction, so this is obviously the better choice. We do this by multiplying the first equation by 2 (the y-coefficient of the other equation) and the second equation by 3 (the y-coefficient of the other equation).

Step 1: (1) × 2 or $2 \times (4x + 3y = 27) \rightarrow 8x + 6y = 54$ (3)

(2) × 3 or $3 \times (5x - 2y = 5) \rightarrow 15x - 6y = 15$ (4)

Label the new equations (3) and (4)

Step 2: Eliminate one of the variables: (3) + (4) $23x = 69$

Step 3: Solve the equation: x = 69 ÷ 23

x = 3

Step 4: Substitute into equation (1) 4(3) + 3y = 27

Step 5: Solve the equation: 12 + 3y = 27

3y = 27 – 12

3y = 15

y = 15 ÷3

y = 5

Step 6: Check: (1), 4 × 3 + 3 × 5 = 12 + 15 = 27, and (2), 5 × 3 – 2 × 5 = 15 – 10 = 5, which are correct so the solution is x = 3 and y = 5.

Consolidation

1. 4x + y = 17 and 2x + y = 9

2. 2x + y = 7 and 5x – y = 14

3. $5x + 2y = 4$ and $4x - y = 11$

4. $3x + 4y = 7$ and $4x + 2y = 1$

5. $2x - 3y = 15$ and $5x + 7y = 52$

6. $2x + 3y = 30$ and $5x + 7y = 71$

7. $2x + 5y = 37$ and $y = 11 - 2x$

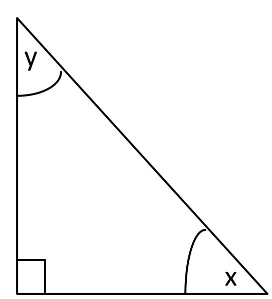
8. $4x - 3y = 7$ and $x = 13 - 3y$

Support Exercise Pg 153 Exercise 10D Nos 1 – 20

Section 8.8 Setting up simultaneous equations

Example 1

One angle in a triangle is 90° and the difference between the other two angles is 36°. Find the larger of the two unknown angles.



Let x be the larger angle.

The sum of the three angles is 180°

Therefore $x + y = 90$ (1)

The difference between x and y is 36°,

Therefore $x - y = 36°$ (2)

The two equations are:

$x + y = 90$ (1)

$x - y = 36$ (2)

$2x = 126$

$x = 126 \div 2$

$x = 63°$

$x + y = 90$

$63 + y = 90$

$y = 90 - 63$

$y = 27°$

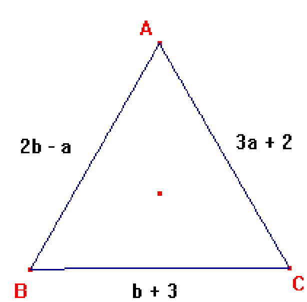
The larger angle is 63°.

Consolidation

1. **The lengths of the sides of an equilateral triangle are (3*a* +2) cm, (2*b* – *a*) cm, and (*b* + 3) cm.**

a) Find *a* and *b*.

b) Find the perimeter of the triangle.



2. **Find two numbers such that twice the first added to the second is 26 and the first added to the second is 28.**

3. **A cup and saucer together cost €2.05. A cup and two saucers cost €2.70. Find the cost of a cup and saucer.**

4. **A rectangle is a cm long and b cm wide. The perimeter of the rectangle is 48cm and the length is 5 cm more than the width. Find the length of the rectangle.**

Support Exercise Pg 154 Exercise 10E Nos 1 – 5