Chapter 12: Quadratic and Cubic Graphs

Section 12.1 Quadratic Graphs

$x^2 + 2$ $a^2 + 2a - 6$ $r - r^2$

$x^2 - 5x + 8$ $2y^2 + 9y + 2$

All the above equations contain a **squared** number.

They are therefore called **quadratic expressions** or **quadratic functions**.

The highest power in a quadratic expression is ***ALWAYS*** 2.

To draw the graphs of quadratic functions we will use the *table of values*.

To draw quadratic graphs, we shall be using the method we used for drawing the straight line graphs.

Points to Remember when Drawing Graphs

When curved graphs are being drawn the following advice should be kept clearly in mind.

1. Do not take too few points. About eight or ten are usually required.
2. To decide where to draw the y-axis look at the range of x-axis, and vice versa.
3. To be able to draw the curve always draw up the table of values in order to work out all the coordinates required.
4. When you draw a smooth curve to pass through the points, always turn the page into a position where your wrist is on the inside of the curve.

Example 1

The most basic quadratic equation is $\mathbf{y = x^2}$.

Draw the graph $\mathbf{y = x^2}$ for the values of x between -3 and 3.

Step 1: Draw the coordinate table

x	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**
$\mathbf{x^2}$	9						
y	**9**						

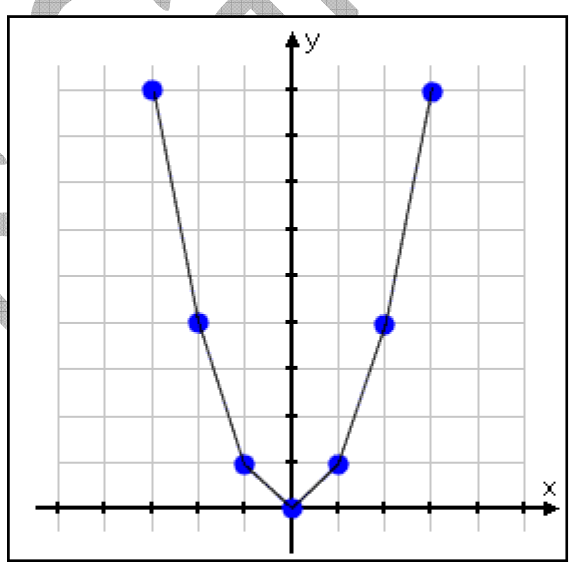
Coordinates: (____,____), (____,____), (____,____), (____,____),

(____,____), (____,____), (____,____)

Step 2: Draw the axis and plot the points

Step 3: Join the points of the curve with a smooth line passing through all the points

The following is the ***incorrect*** way of drawing of a quadratic graph.



The lowest point of the graph is called the **minimum point**.

In the above example the minimum point is at coordinate **(0,0)**.

Example 2

Draw the graph $\mathbf{y = 3x - x^2}$ for the values of x in the range -1 to 4. Draw the graph with the scale of 2cm for one unit on both axes.

x	**-1**	**0**	**1**	**2**	**3**	**4**
3x						
$-x^2$						
y						

Coordinates: (____,____), (____,____), (____,____),

(____,____), (____,____), (____,_____)

a) Find the highest value of this graph and write the value of x in which it occurs. __________

b) Find the values of x where the graph crosses the x-axis. _____________ _____________

The **highest point** of the graph is where the graph turns.

It is called the **maximum point**.

Quadratic graphs always look like $\cap$ or $\cup$.

Example 3

a) Complete this table of values for the graph $\mathbf{y = x^2 + x - 7}$

x	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**
x^2								
x								
-7								
y								

b) Draw the graph $\mathbf{y = x^2 + x - 7}$

c) Write down the values of x where the graph crosses the x-axis

d) Use your graph to find an estimate of the minimum value of the graph you have drawn.

Example 4

Make a table to help you draw the graph $\mathbf{y = x^2 - 3x - 4}$ for the values of x between -2 and 5.

x	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**
$\mathbf{x^2}$								
-3x								
-4								
y								

Use your graph to find

a) the lowest value of $x^2 - 3x - 4$ and the corresponding value of x

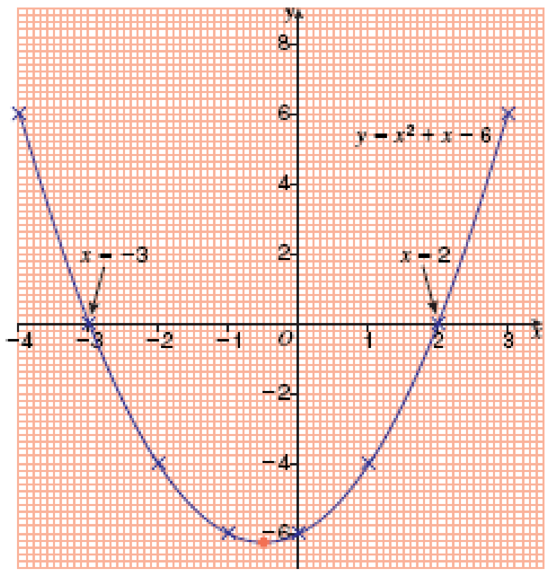
b) the values of x when $x^2 - 3x - 4$ is 0

Support Exercises Pg 382 Exercise 23A Nos 1 – 5

Section 12.2 Using Graphs of Quadratic Functions to solve Equations

The following is the graph for the equation $\mathbf{y = x^2 + x - 6}$

This graph can be used to solve the equation $\mathbf{x^2 + x - 6 = 0}$



The solution of the equation y = $\mathbf{x^2 + x - 6}$ are found by finding the values of x when y = 0.

This is done by observing where the graph crosses the x – axis.

From the graph: when **y = 0**

x = -3 and **x = 2**

Example 1

Draw the graph of the equation $\mathbf{y = 2x^2 - x - 3}$

x	-3	-2	-1	0	1	2	3
$2x^2$							
-x							
-3							
y							

Coordinates: (___,___), (___,___), (___,___), (___,___), (___,___), (___,___), (___,___) From your graph, what are the solutions of the equation **$2x^2 - x - 3 = 0$**?

____

____

Example 2

a) Draw the graph with equation $y = x^2 - 6x - 7$ for values of x between -2 and 8.

b) Give the solutions of the equation $x^2 - 6x - 7 = 0$

a) Draw the coordinate table to retrieve the coordinates of the graph

x	-2	-1	0	2	3	4	5	6	7	8
x^2										
-6x										
-7										
y										

Coordinates:

(___,___), (___,___), (___,___), (___,___), (___,___), (___,___),

(___,___), (___,___), (___,___), (___,___)

From the graphs, what are the solutions of the equation **$x^2 - 6x - 7 = 0$**?

____

____

Example 3

Complete the table for the graph $\mathbf{y = x^2 - 2x - 8}$ for the values of x from -3 to 5. Use your table to draw the graph.

x	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**
$\mathbf{x^2}$									
-2x									
-8									
y									

a) Use your graph to find the value of y when x = 0.5 ______________________________

b) Use your graph to solve the equation $x^2 - 2x - 8 = 3$ ______________________________

Example 4

a) Complete the table to draw the graph of $\mathbf{y = x^2 - 6x + 3}$ for the values of x between -1 and 7

x	**-1**	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
$\mathbf{x^2}$									
-6x									
+3									
y									

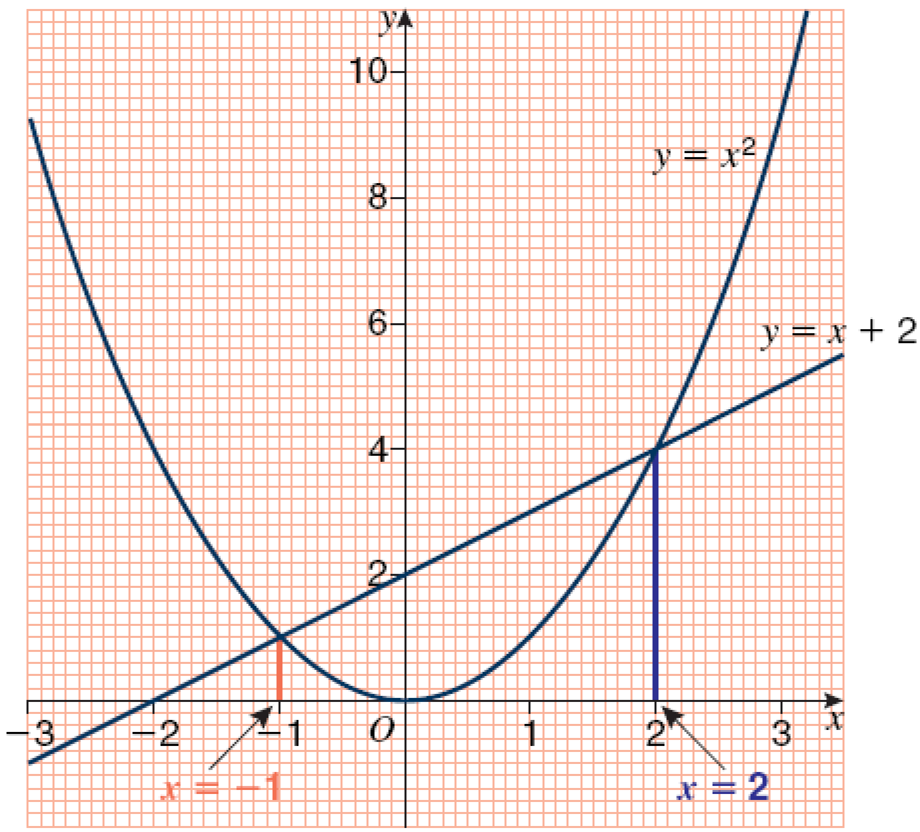
b) Where does the graph intersect with the x-axis ___________________________________

c) Use your graph to find the y-value when x = 3.5 ___________________________________

d) Use your graph to solve the equation $x^2 - 6x + 3 = 5$ ___________________________________

Support Exercise Pg 384 Exercise 23B Nos 1, 2, 3, 4 & 5

Section 12.3 Using Graphs of Quadratic and Linear Graphs to solve Quadratic Equations



The diagram shows the graph of $y = x^2$ and $y = x + 2$

The graphs cross at **TWO** points.

The coordinates of these two points are (-1, 1) and (2, 4)

These coordinates satisfy both equations simultaneously because

$1 = (-1)^2$	and	$1 = -1 + 2$
$4 = (2)^2$	and	$4 = 2 + 2$

Eliminating y from $y = x^2$ and $y = x + 2$ gives the equation

$x^2 = x + 2$ or $x^2 - x - 2 = 0$

From the graph the solution of the quadratic equation $\mathbf{x^2 - x - 2 = 0}$ are x = -1 and x = 2

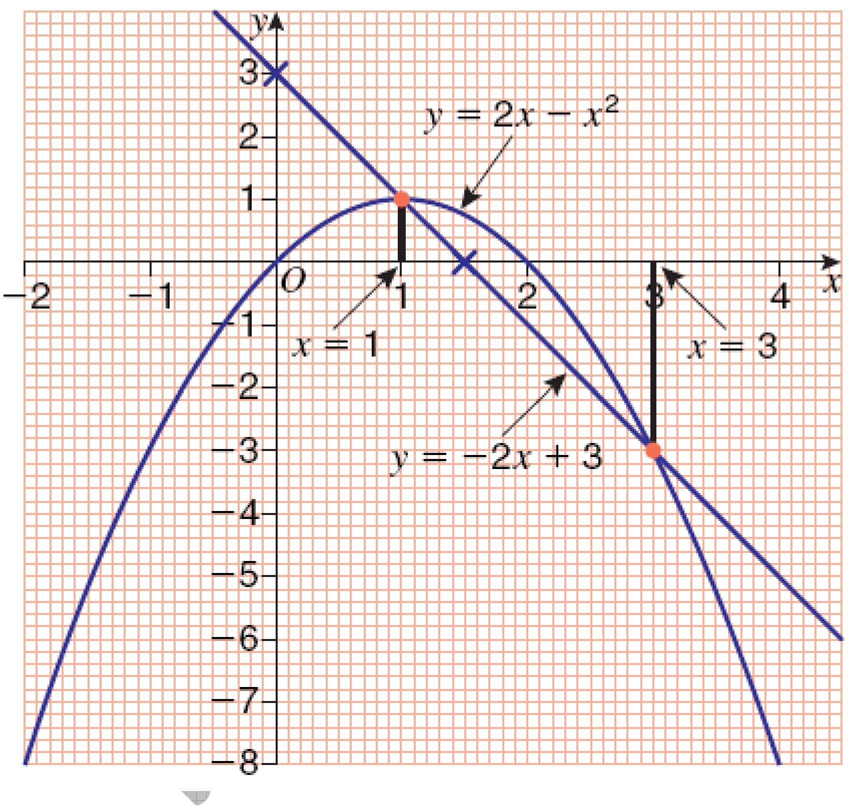
Example 1

The diagram shows the graph of $\mathbf{y = 2x - x^2}$

By drawing a suitable straight line on the graph estimate the solutions of the equation $\mathbf{x^2 - 4x + 3 = 0}$

$x^2 - 4x + 3 = 0$	*[The equation cannot be directly compared so we must REARRANGE]*
$3 = -x^2 + 4x$	*[Subtract x2 from both sides and add 4x]*
$3 = -x^2 + 2x + 2x$	*[Write 4x as 2x + 2x as we need 2x to compare to $y = 2x - x^2$]*
$-2x + 3 = 2x - x^2$	*[The equations can be compared now]*

Comparing $-2x + 3 = 2x - x^2$ with the graph $y = 2x - x^2$ gives $y = -2x + 3$



Solution : x = 1 and x = 3

Example 2

Draw the graph of **$y = 2x^2 + 3x - 6$** for the values of x from -4 to 2, taking 2cm to represent 1 unit on the x-axis and 1cm to represent I unit in the y-axis. Use the graph to solve the following equations:

a) $2x^2 + 3x - 6 = 0$

b) $2x^2 + 3x - 11 = 0$

c) $2x^2 + 3x + 3 = 0$

d) $3 - 3x - 2x^2 = 0$

Example 3

Using the same scales and axes, draw the graphs of **$y = x^2 - 4x + 7$** and **$y = x + 1$** for values of x from 0 to 5, taking 2 cm to represent 1 unit both axes. From your graph find,

a) the values of x where the 2 graphs intersect.

b) the equation in x whose roots are the points of intersection of the 2 graphs.

c) the solutions of the equations $x^2 - 4x + 2 = 0$

Support Exercise Pg 388 Exercise 23C Nos 1 - 6

Section 12.4 Cubic Graphs

A **cubic** function or graph is one that contains a term in x^3. The following are examples of cubic graphs.

$y = x^3$ $y = x^3 + 3x$ $y = x^3 + x^2 + x + 1$

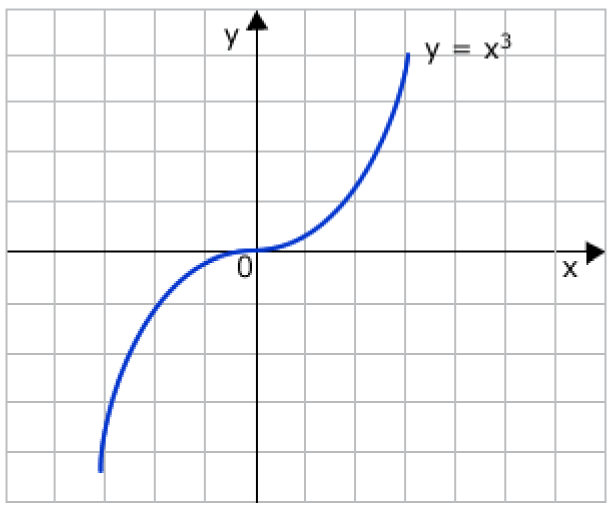
The techniques to draw them are exactly the same as those for quadratic graphs.

Example 1

Plot the graph $\mathbf{y = x^3}$ for the values of x between -2 to 2 using half intervals.

x	**-2**	**-1.5**	**-1**	**-0.5**	**0**	**0.5**	**1**	**1.5**	**2**
x^3			1						
y									

Coordinates: (___,___) (___,___) (___,___) (___,___) (___,___) (___,___) (___,___)
(___,___) (___,___)



Example 2

a) Complete the table of values for the graph $\mathbf{y = x^3 - 2x^2}$

x	**-2**	**-1**	**0**	**1**	**2**	**3**
x^3					8	
$-2x^2$					-8	
y					**0**	

b) Use your graph to write down the solutions to 1 d.p. of

i. $x^3 - 2x^2 = 4$ ii. $x^3 - 2x^2 = -1$ iii. $x^3 - 2x^2 = -3$

c) What line could you draw on your graph to solve $x^3 - 2x^2 = x$? _______

Draw the line and find the solution to 1d.p.

d) Use your graph to estimate the solution to $x^3 - 2x^2 = 1$

Example 3

Draw the graph $\mathbf{y = x^3 - 3x^2 - x + 3}$ for the values of x from -2 to 4.

x	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**
x^3							
$-3x^2$							
-x							
+3							
y							

a) From your graph to solve $x^3 - 3x^2 - x + 3 = 0$

b) Find the maximum and minimum values

Example 4

Complete the table to draw the graph of $\mathbf{y = x^3 + 2x + 5}$ for the values of x between -2 and 2

x	**-2**	**-1.5**	**-1**	**-0.5**	**0**	**0.5**	**1**	**1.5**	**2**
$\mathbf{x^3}$									
2x									
+5									
y									

Use your graph to solve the equation x3 – 2x + 5 = 3.

Support Exercise Pg 416 Exercise 25A Nos 2 & 3

Section 12.5 Reciprocal Graphs

A reciprocal equation has the form $y = \frac{a}{x}$

Examples of reciprocal equations are:

$y = \frac{1}{x}$ $y = \frac{4}{x}$ $y = -\frac{3}{x}$

All reciprocal graphs have a similar shape and some symmetry properties.

Example 1

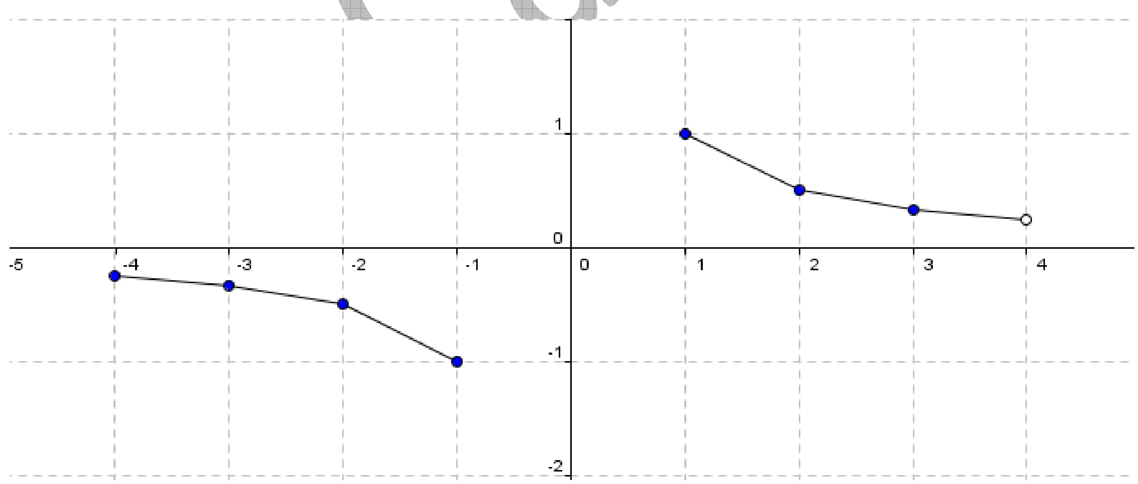
Complete the table to draw the graph of $y = \frac{1}{x}$ for $-4 \leq x \leq 4$.

Values are rounded to two decimal places, as it is unlikely that you could plot a value more accurately than this.

x	**-4**	**-3**	**-2**	**-1**	**1**	**2**	**3**	**4**
$^1/_x$	-0.25	-0.33	-0.5	-1	1	0.5	0.33	0.25
y	**-0.25**	**-0.33**	**-0.5**	**-1**	**1**	**0.5**	**0.33**	**0.25**

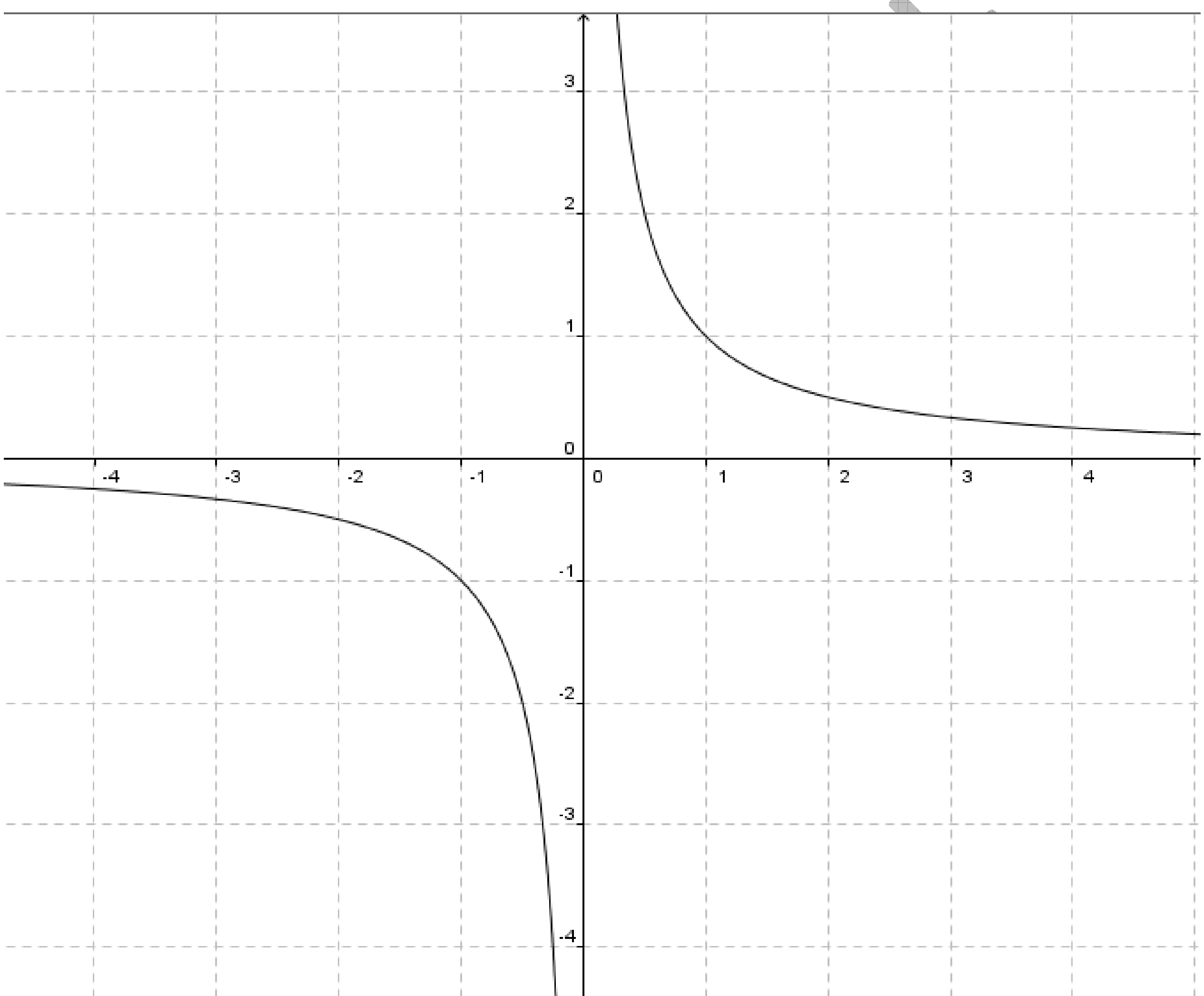
The graph plotted from the above coordinates is shown below. This is not much of a graph and does not show the properties of a reciprocal function. If you take x-values from -0.8 and 0.8 in steps of 0.2, you get the next table.

Notes that you cannot use x = 0 since $^1/_0$ is infinity (∞).



x	-0.8	-0.6	-0.4	-0.2	0.2	0.4	0.6	0.8
$^1/_x$	-1.25	-1.67	-2.5	-5	5	2.5	1.67	1.25
y	**-1.25**	**-1.67**	**-2.5**	**-5**	**5**	**2.5**	**1.67**	**1.25**

If we plot these as well we will have the following graph.

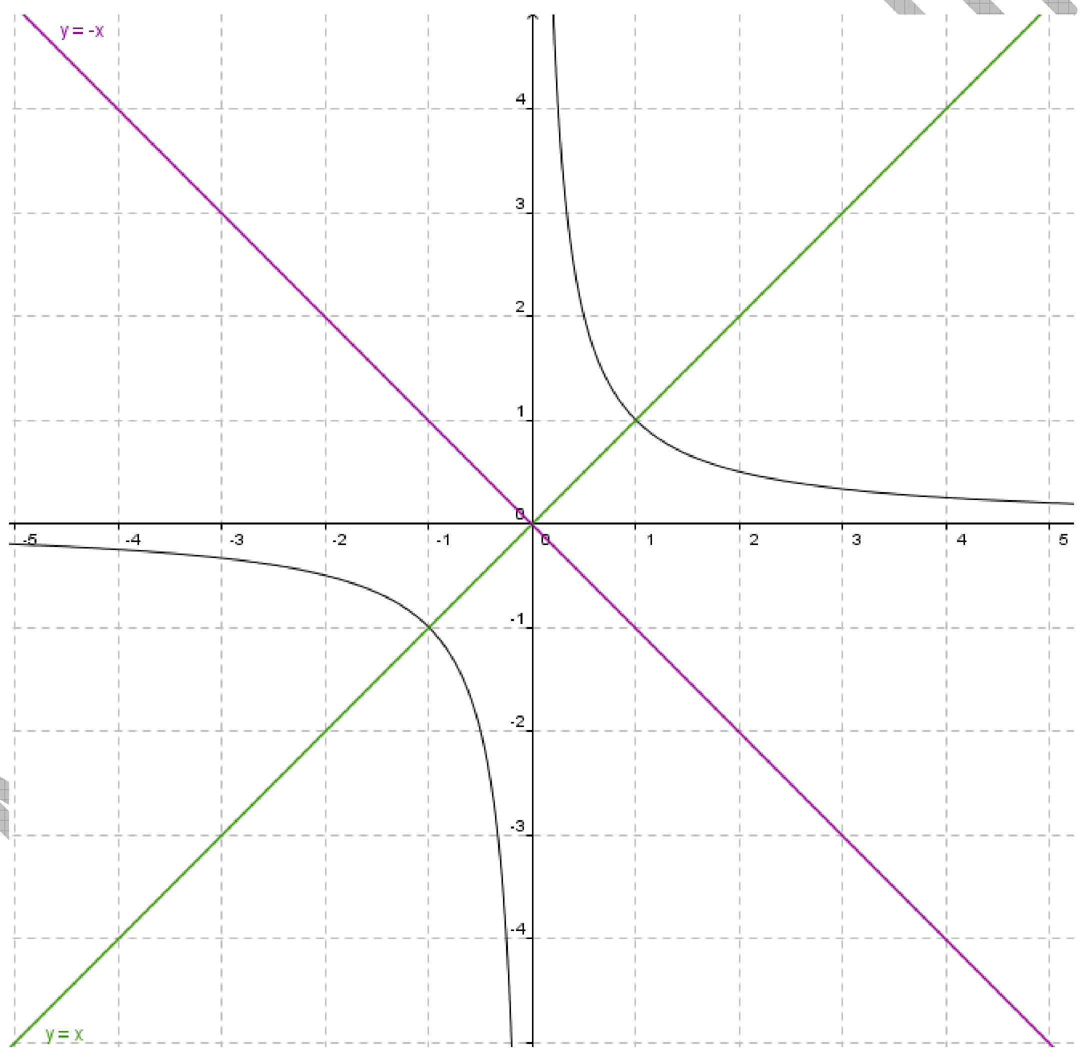


From the graph above, the following properties can be seen.

- The lines **y = x** and **y = -x** are lines of symmetry
- The closer x gets to **zero**, the nearer the graph gets to the y-axis.
- As x increases, the graph gets closer to the x-axis

The graph never actually touches the axes, it just gets closer and closer to the.

These properties are true for all *reciprocal* graphs.



Example 2

Complete the table to draw the graph $y = \frac{2}{x}$ for -4 ≤ x ≤ 4.

x	0.2	0.4	0.5	0.8	1	1.5	2	3	4
$^2/_x$									
y									

a) Use your graph to find the y-value when x = 2.5.

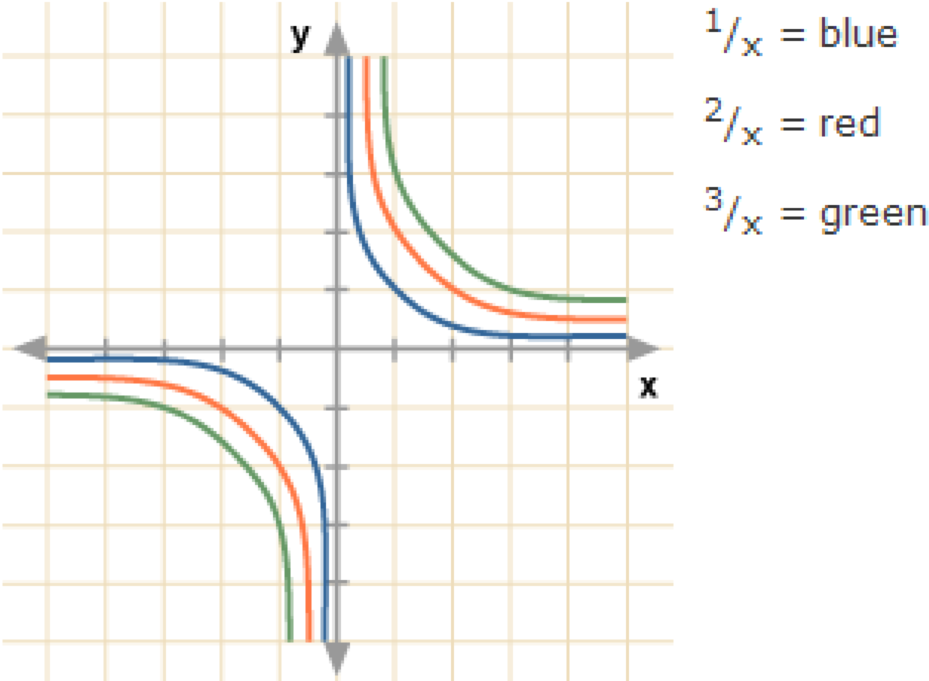
b) Use your graph to solve the equation $\frac{2}{x} = 7$

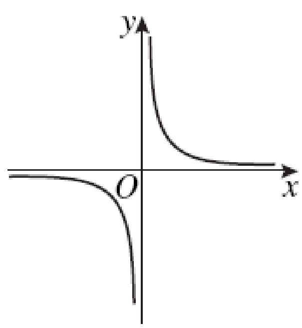
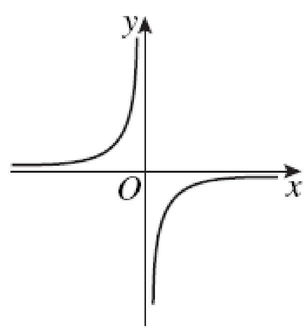
c) Use your graph to solve the equation $\frac{2}{x} = -1.25$

Example 3

Complete the table to draw the graph $y = -\frac{2}{x}$ for -4 ≤ x ≤ 4.

x	0.2	0.4	0.5	0.8	1	1.5	2	3	4
$^{-2}/_x$									
y									



Reciprocals $y = \frac{k}{x}$	 $k > 0$  $k < 0$

Support Exercise Pg 416 Exercise 25A Nos 5 and 6