Chapter 12: Quadratic and Cubic Graphs

Section 12.1 Quadratic Graphs

$a^2 + 2a - 6$

$x^2 + 2$ $\quad$ $r - r^2$

$x^2 - 5x + 8$ $\quad$ $2y^2 + 9y + 2$

All the above equations contain a **squared** number.

They are therefore called **quadratic expressions** or **quadratic functions**.

The highest power in a quadratic expression is ***ALWAYS*** 2.

To draw the graphs of quadratic functions we will use the *table of values*.

To draw quadratic graphs, we shall be using the method we used for drawing the straight line graphs.

Points to Remember when Drawing Graphs

When curved graphs are being drawn the following advice should be kept clearly in mind.

1. Do not take too few points. About eight or ten are usually required.
2. To decide where to draw the y-axis look at the range of x-axis, and vice versa.
3. To be able to draw the curve always draw up the table of values in order to work out all the coordinates required.
4. When you draw a smooth curve to pass through the points, always turn the page into a position where your wrist is on the inside of the curve.

Ms Camenzuli

Example 1

The most basic quadratic equation is $\mathbf{y = x^2}$.

Draw the graph $\mathbf{y = x^2}$ for the values of x between -3 and 3.

Step 1: Draw the coordinate table

x	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**
$\mathbf{x^2}$	9						
y	**9**						

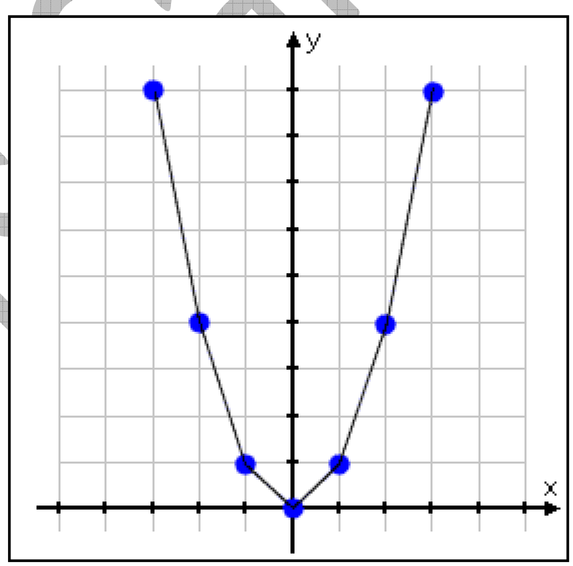
Coordinates: (____,____), (____,____), (____,____), (____,____),

(____,____), (____,____), (____,_____)

Step 2: Draw the axis and plot the points

Step 3: Join the points of the curve with a smooth line passing through all the points

The following is the ***incorrect*** way of drawing of a quadratic graph.



The lowest point of the graph is called the **minimum point**.

In the above example the minimum point is at coordinate **(0,0)**.

Example 2

Draw the graph $y = 3x - x^2$ for the values of x in the range -1 to 4. Draw the graph with the scale of 2cm for one unit on both axes.

x	**-1**	**0**	**1**	**2**	**3**	**4**
3x						
$-x^2$						
y						

<u>*Coordinates*</u>: (____,____), (____,____), (____,____),

(____,____), (____,____), (____,_____)

a) Find the highest value of this graph and write the value of x in which it occurs. ___________

b) Find the values of x where the graph crosses the x-axis. ______________ ______________

The **<u>highest point</u>** of the graph is where the graph turns.

It is called the **maximum point**.

Quadratic graphs always look like ∩ or ∪.

Example 3

a) Complete this table of values for the graph $y = x^2 + x - 7$

x	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**
x^2								
x								
-7								
y								

Ms Camenzuli

b) Draw the graph $\mathbf{y = x^2 + x - 7}$

c) Write down the values of x where the graph crosses the x-axis

d) Use your graph to find an estimate of the minimum value of the graph you have drawn.

Example 4

Make a table to help you draw the graph $\mathbf{y = x^2 - 3x - 4}$ for the values of x between -2 and 5.

x	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**	**5**
x^2								
-3x								
-4								
y								

Use your graph to find

a) the lowest value of $x^2 - 3x - 4$ and the corresponding value of x

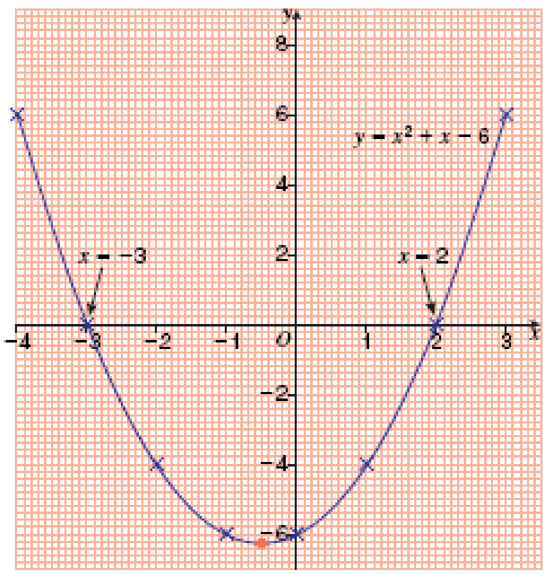
b) the values of x when $x^2 - 3x - 4$ is 0

Support Exercises Pg 382 Exercise 23A Nos 1 – 5

Ms Camenzuli

The following is the graph for the equation **$y = x^2 + x - 6$**

This graph can be used to solve the equation **$x^2 + x - 6 = 0$**



The solution of the equation y = **$x^2 + x - 6$** are found by finding the values of x when y = 0.

This is done by observing where the graph crosses the x – axis.

From the graph: when **y = 0**

x = -3 and **x = 2**

Example 1

Draw the graph of the equation **$y = 2x^2 - x - 3$**

x	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**
$2x^2$							
-x							
-3							
y							

Coordinates: (___,___), (___,___), (___,___), (___,___), (___,___), (___,___), (___,___)

Ms Camenzuli

From your graph, what are the solutions of the equation $\mathbf{2x^2 - x - 3 = 0}$?

___

___

Example 2

a) Draw the graph with equation $y = x^2 - 6x - 7$ for values of x between -2 and 8.

b) Give the solutions of the equation $x^2 - 6x - 7 = 0$

a) Draw the coordinate table to retrieve the coordinates of the graph

x	-2	-1	0	2	3	4	5	6	7	8
x^2										
-6x										
-7										
y										

Coordinates:

(___,___), (___,___), (___,___), (___,___), (___,___), (___,___),

(___,___), (___,___), (___,___), (___,___)

From the graphs, what are the solutions of the equation $\mathbf{x^2 - 6x - 7 = 0}$?

___

___

Ms Camenzuli

Example 3

Complete the table for the graph $y = x^2 - 2x - 8$ for the values of x from -3 to 5. Use your table to draw the graph.

x	-3	-2	-1	0	1	2	3	4	5
x^2									
-2x									
-8									
y									

a) Use your graph to find the value of y when x = 0.5 ___________________________

b) Use your graph to solve the equation $x^2 - 2x - 8 = 3$ ___________________________

Example 4

a) Complete the table to draw the graph of $y = x^2 - 6x + 3$ for the values of x between -1 and 7

x	-1	0	1	2	3	4	5	6	7
x^2									
-6x									
+3									
y									

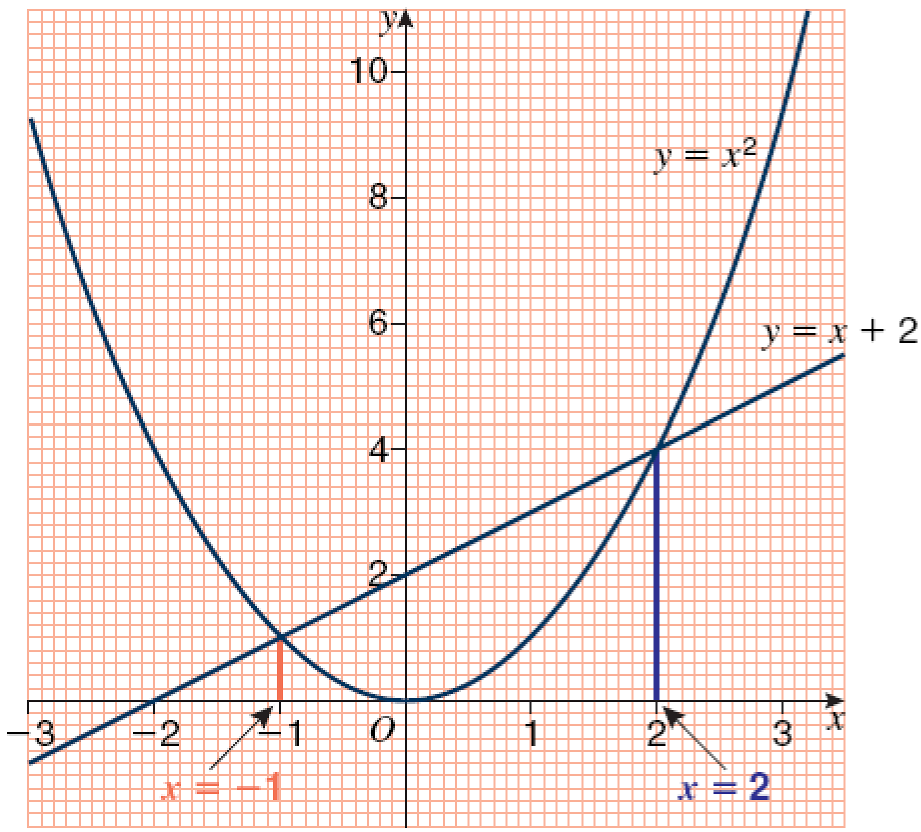
b) Where does the graph intersect with the x-axis ________________________________

c) Use your graph to find the y-value when x = 3.5 ________________________________

d) Use your graph to solve the equation $x^2 - 6x + 3 = 5$ ________________________________

Support Exercise Pg 384 Exercise 23B Nos 1, 2, 3, 4 & 5

Ms Camenzuli



The diagram shows the graph of $y = x^2$ and $y = x + 2$

The graphs cross at **TWO** points.

The coordinates of these two points are (-1, 1) and (2, 4)

These coordinates satisfy both equations simultaneously because

$1 = (-1)^2$	and	$1 = -1 + 2$
$4 = (2)^2$	and	$4 = 2 + 2$

Eliminating y from $y = x^2$ and $y = x + 2$ gives the equation

$x^2 = x + 2$ or $x^2 - x - 2 = 0$

From the graph the solution of the quadratic equation $\mathbf{x^2 - x - 2 = 0}$ are x = -1 and x = 2

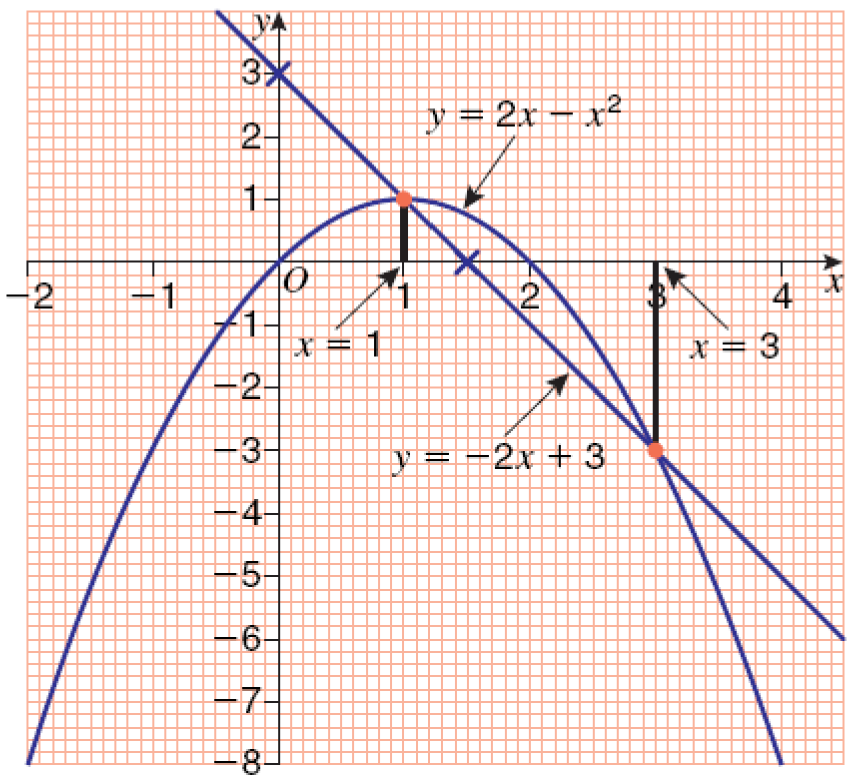
Example 1

The diagram shows the graph of $\mathbf{y = 2x - x^2}$

By drawing a suitable straight line on the graph estimate the solutions of the equation $\mathbf{x^2 - 4x + 3 = 0}$

$x^2 - 4x + 3 = 0$	*[The equation cannot be directly compared so we must REARRANGE]*
$3 = -x^2 + 4x$	*[Subtract x2 from both sides and add 4x]*
$3 = -x^2 + 2x + 2x$	*[Write 4x as 2x + 2x as we need 2x to compare to $y = 2x - x^2$]*
$-2x + 3 = 2x - x^2$	*[The equations can be compared now]*

Comparing $-2x + 3 = 2x - x^2$ with the graph $y = 2x - x^2$ gives $y = -2x + 3$



Solution : x = 1 and x = 3

Example 2

Draw the graph of $\mathbf{y = 2x^2 + 3x - 6}$ for the values of x from -4 to 2, taking 2cm to represent 1 unit on the x-axis and 1cm to represent I unit in the y-axis. Use the graph to solve the following equations:

a) $2x^2 + 3x - 6 = 0$

b) $2x^2 + 3x - 11 = 0$

c) $2x^2 + 3x + 3 = 0$

d) $3 - 3x - 2x^2 = 0$

Example 3

Using the same scales and axes, draw the graphs of $\mathbf{y = x^2 - 4x + 7}$ and $\mathbf{y = x + 1}$ for values of x from 0 to 5, taking 2 cm to represent 1 unit both axes. From your graph find,

a) the values of x where the 2 graphs intersect.

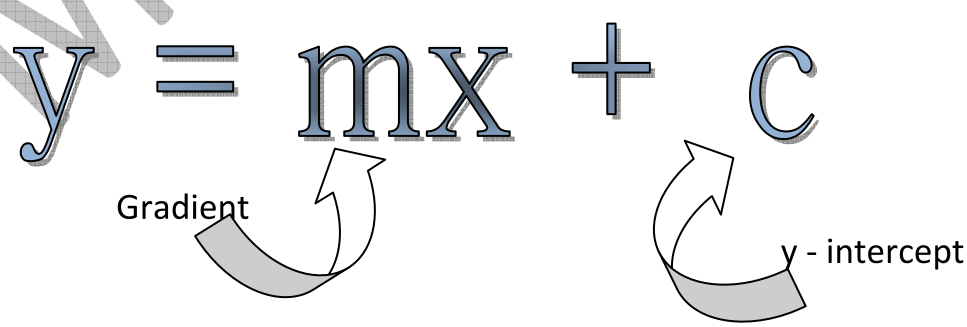
b) the equation in x whose roots are the points of intersection of the 2 graphs.

c) the solutions of the equations $x^2 - 4x + 2 = 0$

Support Exercise Pg 388 Exercise 23C Nos 1 - 6

Section 12.4 Using the equation y = mx + c

The general equation of a line is:



Ms Camenzuli

Example 1

Write the equation of a line with gradient 4 and y-intercept 7.

y = mx + c

m = 4

c = 7

Answer: **y = 4x + 7**

Example 2

Write down the gradient, m, and y intercept, c, for the given equations.

y = ½ x – 4

m = ½

c = -4

Consolidation

Complete the following table.

Equation	**Gradient**	**Y – intercept**
y = -3 – 7x		
	¾	7
	0	2
3y = 9x – 12		
	-5	-1
y + 3x – 4 = 0		
y = -5x		

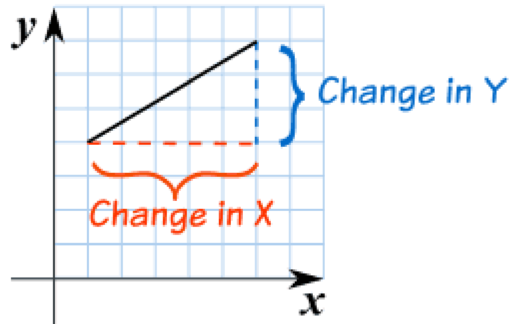
Support Exercise Pg 205 Exercise 13D Nos 6 - 11

Ms Camenzuli

The method to calculate the Gradient is:

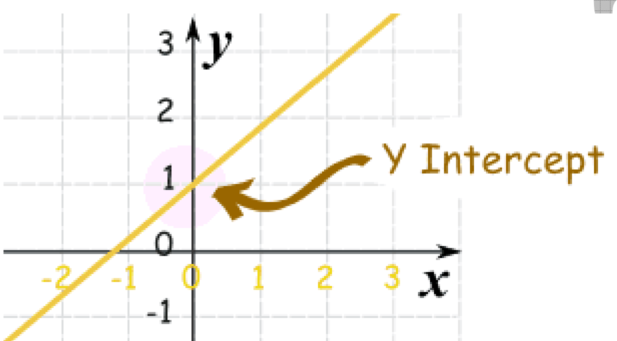
Divide the **change in height** by the **change in horizontal distance**

$$\textbf{Gradient} = \frac{\text{Change in Y}}{\text{Change in X}}$$



The y-intercept

The Y intercept of a straight line is simply where the line crosses the Y axis.



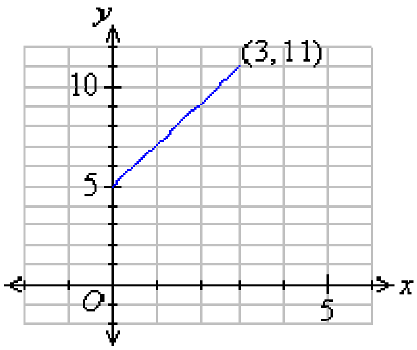
In the above diagram the line crosses the Y axis at 1.

The Y intercept is equal to 1.

Ms ...

Example 1

Calculate the gradient of the straight line given in the following diagram; and find its equation.



Solution:

Let $(x_1, y_1) = (0, 5)$ and $(x_2, y_2) = (3, 11)$.

$\therefore m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{11-5}{3-0} = \dfrac{6}{3} = 2$

$c = 5$

The general equation of the straight line is

$y = mx + c$

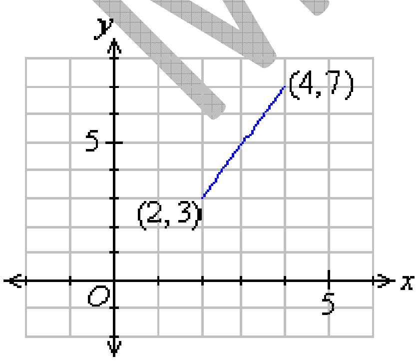
Substituting $m = 2$ and $c = 5$ gives

$y = 2x + 5$

Example 2

Find the equation of the line joining the points (2, 3) and (4,7).

Solution:



Ms Camenzuli

Let $(x_1, y_1) = (2, 3)$ and $(x_2, y_2) = (4, 7)$.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{7-3}{4-2}$$
$$= \frac{4}{2}$$
$$= 2$$

The general equation of a straight line is

$$y = mx + c$$
$$\therefore y = 2x + c$$

Point $(2, 3)$ is on the line.

$$\therefore 3 = 2 \times 2 + c$$
$$3 = 4 + c$$
$$3 - 4 = c + 4 - 4$$
$$\therefore c = -1$$

So, the equation is $y = 2x - 1$.

Consolidation

Find the equations of the lines passing through the following points.

1) (5,1) and (7,9)

2) (1,2) and (6,-7)

Ms Camenzuli

3) (-2,4) and (2,1)

4) (-9,-3) and (6,0)

5) (4,-3) and (2,-7)

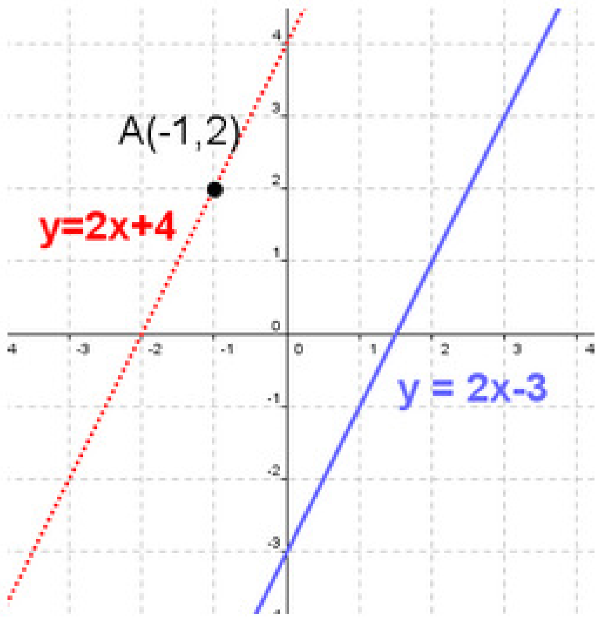
6) (3,1) and (5,-2)

Ms Camenzuli

Support Exercise Pg 209 Ex 13E Nos 1 – 3, 10 – 12

For two lines to be parallel, they must never meet. For two graphs never to meet the inclination of each line must be the same. Therefore, two parallel lines must always have the SAME gradient but DIFFERENT y-intercepts.

Parallel lines have the SAME gradient



Example 1

Find the equation of the line parallel to $y = 3x - 1$ that passes through the point (0, 5).

Solution: As the line is parallel to $y = 3x - 1$, it must have the same gradient, i.e. 3.

As our line must pass through (0, 5), the y-intercept is 5.

So the required equation is $\boldsymbol{y = 3x + 5}$.

Example 2

Find the equation of the line parallel to *y* = 8 – 2*x* passing through the point (3, 7).

Solution: A parallel line has the same gradient i.e. -2.

The equation of the parallel line therefore is $y = -2x + c$.

In order to find c, we can use the coordinates of the point that we wish our line to pass through.

Substituting $x = 3$, $y = 7$ gives:

$7 = -2 \times 3 + c$

$c = 7 + 6 = 13$.

So the equation is $\boldsymbol{y = -2x + 13}$.

Consolidation

1) Find, in the form y = mx + c, the equation of the straight line parallel to the line y = 4x − 1 which passes through the point with coordinates (1, 7)

2) The straight line l passes through the points A (−5, 5) and B (1, 7).

   a) Find an equation of the line l. Give your answer in the form y = mx + c.

Ms Camenzuli

b) Find the equation of a line parallel to l and that passes through the point (3,2)

3) Find the equation of a line parallel to the line $2x - 3y + 5 = 0$ which passes through the point with coordinates $(3, -1)$

Support Exercise Pg 209 Ex 13E Nos 4 – 6, 13

Ms Camenzuli