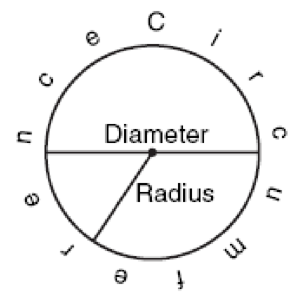
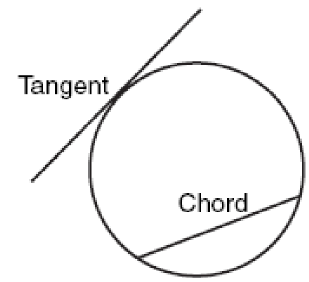
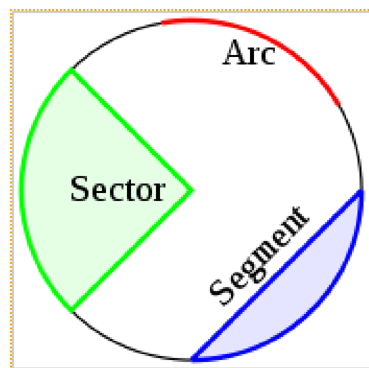
Chapter 3 Circles Theorem

Section 3.1 Circle Properties





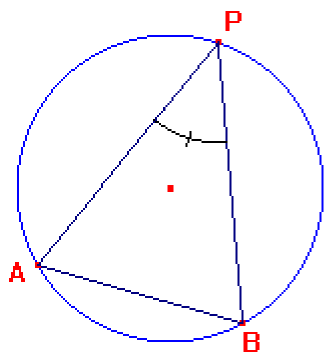
- The **CIRCUMFERENCE** is the distance around the edge of a circle.
- A **CHORD** is a straight line segment joining two points on a circle.



- A **DIAMETER** is a chord that passes through the centre of a circle.
- A **RADIUS** is the distance from the centre of a circle to a point on the circle.
- A **TANGENT** is a line that touches the circle at only one point.
- An **ARC** of a circle is any part of the circle's circumference.
- A **SECTOR** is a region bounded by two radii and an arc lying between the radii.
- A **SEGMENT** is a region bounded by a chord and an arc lying between the chord's endpoints

Section 3.2 Theorem 1 – The angle subtended by the arc at the centre of the circle is twice the angle subtended at the circumference

Subtended Angles

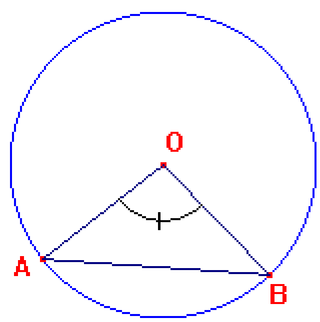


If A, B and P are three points on the circumference of a circle with centre O, then we say that

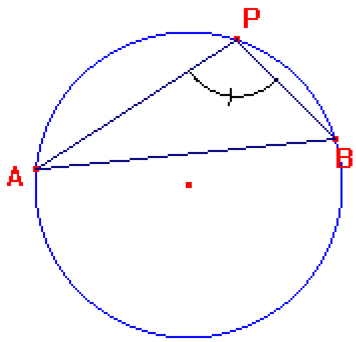
Angle APB is subtended either by the arc AB or by chord AB

Alternatively we can say that

both the arc AB and the chord AB subtend the angle APB at the circumference

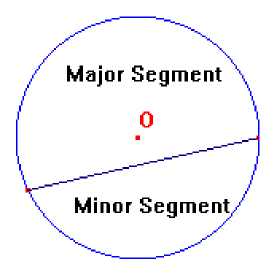


The arc and chord also subtend the angle AOB at the ***centre***.



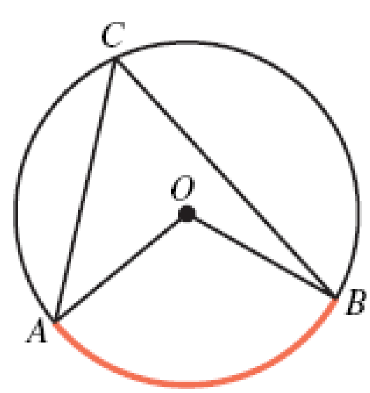
Note that a chord can subtend an obtuse angle at the circumference.

In such a case the angle is in the minor segment.



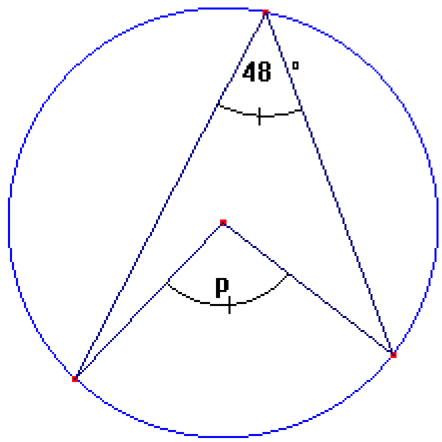
The angle subtended by an arc at the centre of a circle is twice the angle subtended at the circumference

Angle AOB = 2 × ACB



The angle subtended at the centre of a circle by an arc is twice any angle subtended at the circumference by the same arc

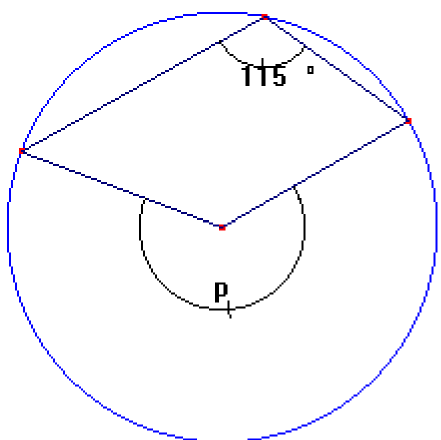
<u>Example 1</u>



p = 2 x 48

p = 96 (∠ at centre = 2 x ∠ at circ)

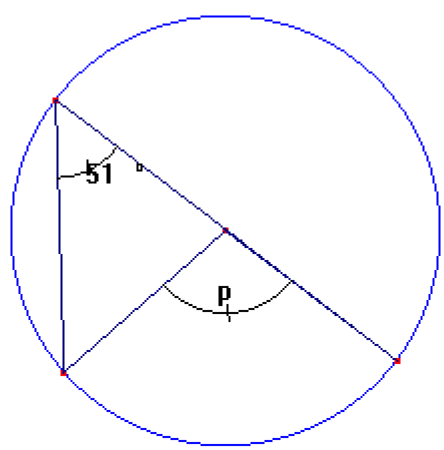
<u>Example 2</u>



p = 2 x 115

p = 230 (∠ at centre = 2 x ∠ at circ)

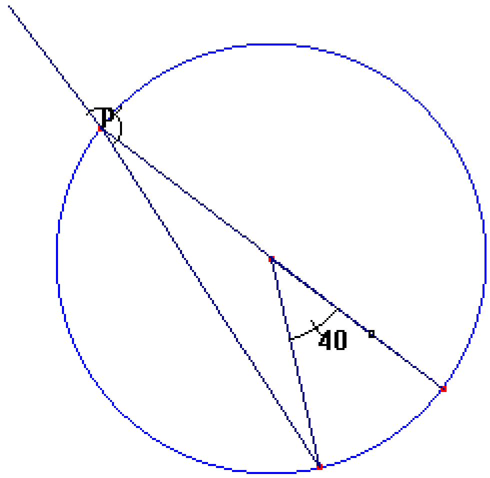
<u>Example 3</u>



p = 2 x 51

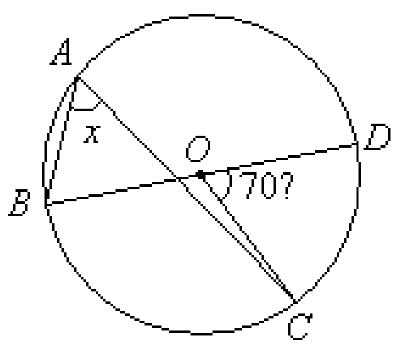
p = 102 (∠ at centre = 2 x ∠ at circ)

Example 4 **Find the value of angle p.**



Example 5

Find the value of angle x.



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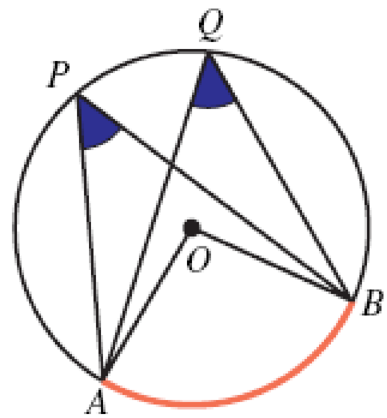
Section 3.3Theorem 2: Angles in the same segment are equal

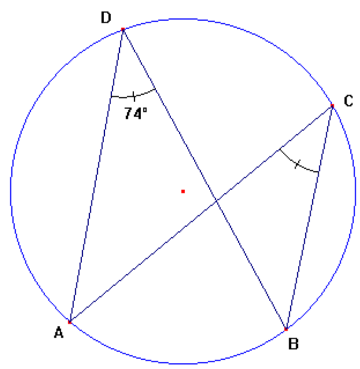
Angles subtended by the same arc are equal.

Angle APB = Angle AQB



Example 1

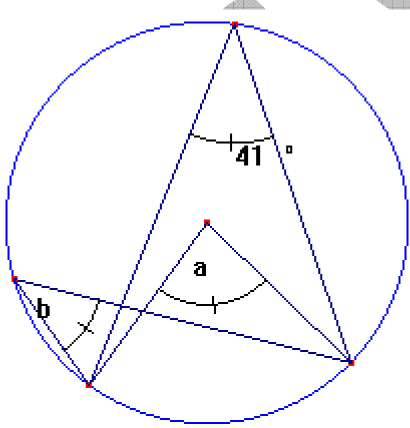
Find the missing angles giving reasons for your answer.



Angle ACB = 74 (Ls subtended by the same chord)

Example 2

Find the missing angles and give reasons for your answer.

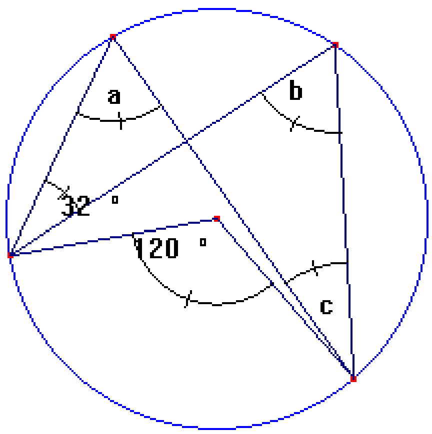


a = 41 x 2 = 82 (∠ at centre = 2 x ∠ at circ)

b = 41 (Ls subtended by the same chord)

Example 3

Find the missing angles and give reasons for your answer.



a = 120 / 2 = 60 (∠ at centre = 2 x ∠ at circ)

b = 60 (Ls sub by the same chord)

c = 32 (Ls sub by the same chord)

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Handout

Section 3.4 Theorem 3: The angle subtended by the diameter is a right angle

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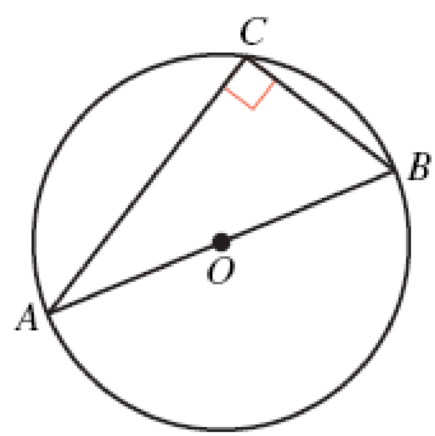
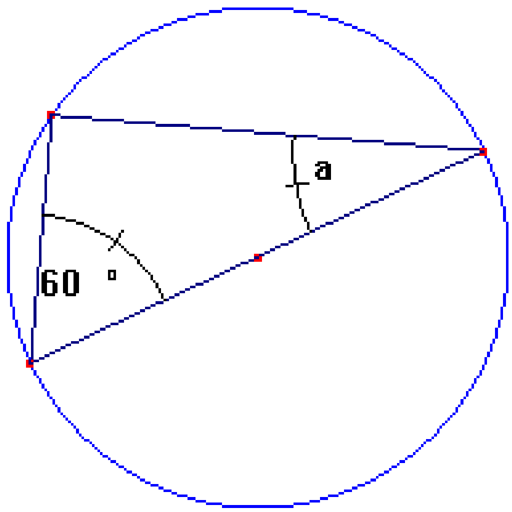
The angle subtended by the diameter is 90°.

Angle ACB = 90°

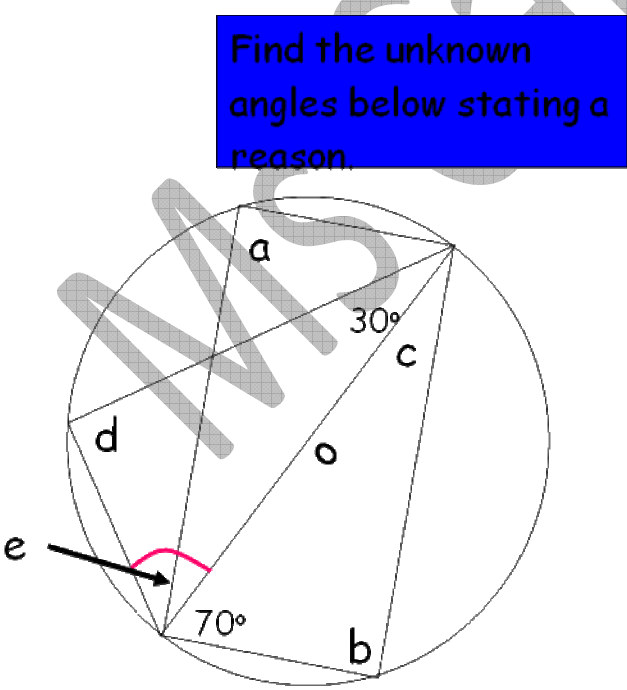


This can be derived from the previous theorem. Since the angle at the centre (180) is twice the angle at the circumference (90) we can say that the angle at the circumference is a right angle.

Example 1



Example 2



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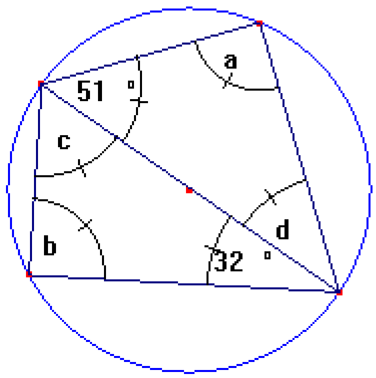
Example 3

Find the unknown angles

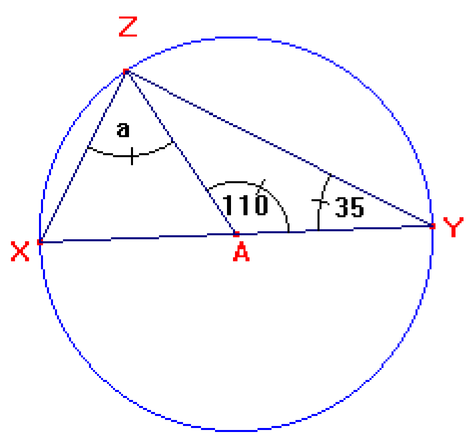


Example 4

In the diagram XY is a diameter of the circle and ∠AZX is *a*.

Ben says that the value of *a* is 50°.

Give reasons to explain why he is wrong.



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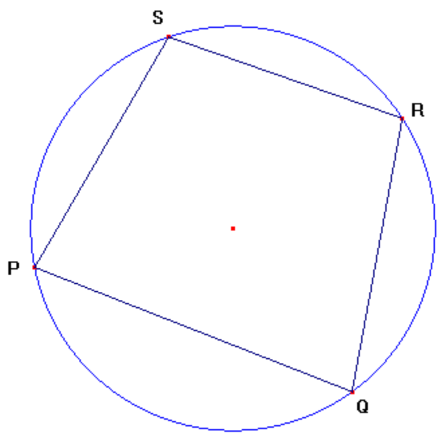
Section 3.5 Theorem 4: The sum of the opposite angles of a cyclic quadrilateral is 180°

A quadrilateral whose vertices (corners) all lie on the circumference of a circle is called a **cyclic quadrilateral.**



Angle SPQ + Angle SRQ = **180°**

And

Angle PSR + Angle PQR = **180°**

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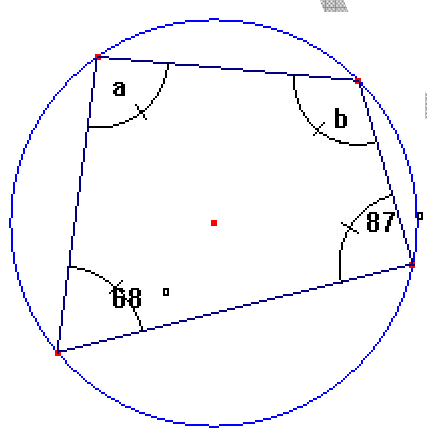
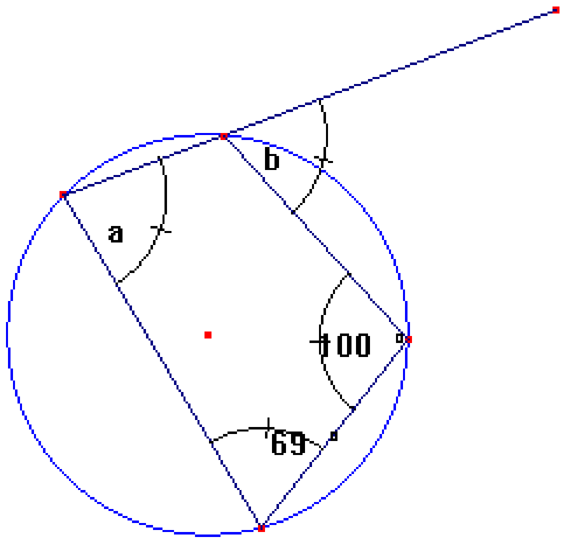
Example 1

Find the missing angle giving a reason for your answer.



Example 2

Find the missing angle giving a reason for your answer.



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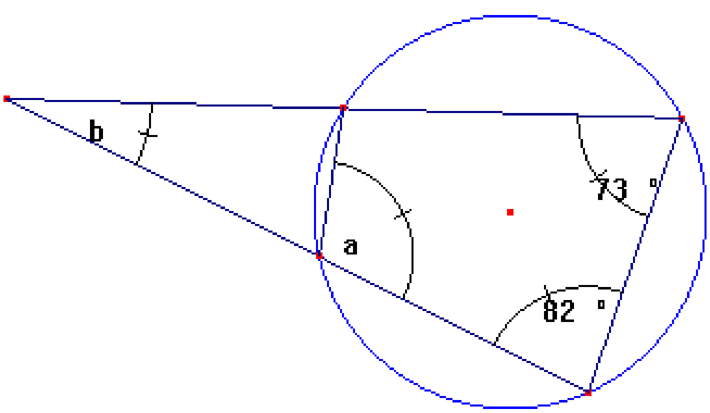
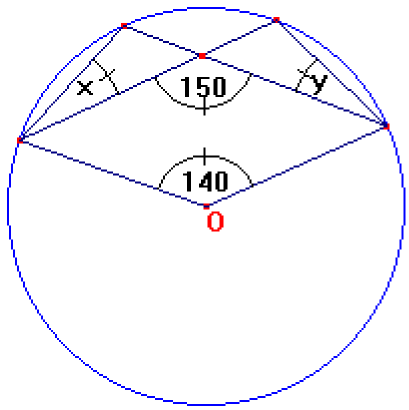
Example 3

Find the missing angle giving a reason for your answer.



Example 4

Find the angles marked in letters.



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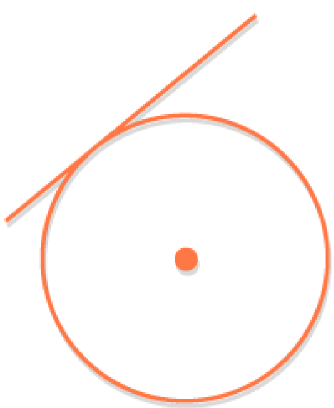
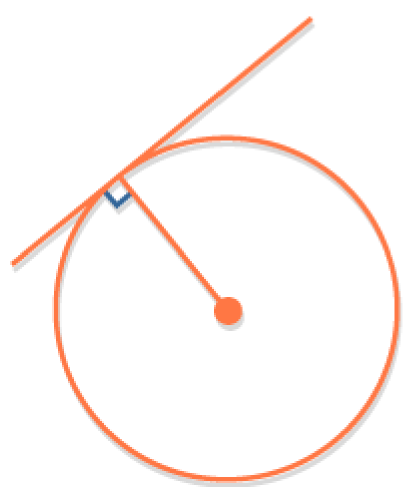
Handout

Section 3.6 Theorem 5: The angle between a radius and tangent form a right angle



A **tangent** to a circle is a line which just touches the circle.



Remember:

A tangent is always at right angles to the radius where it touches the circle.

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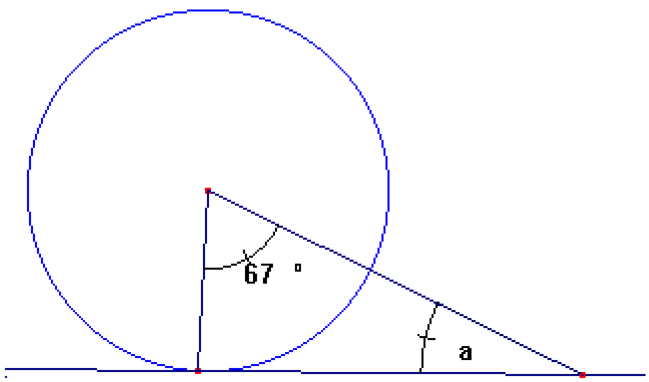
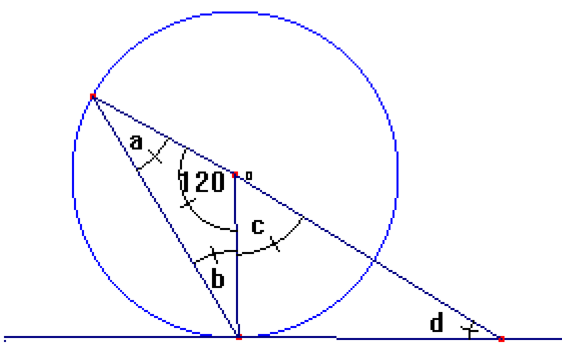
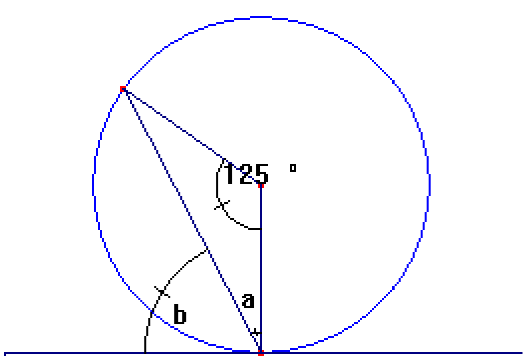
Example 1



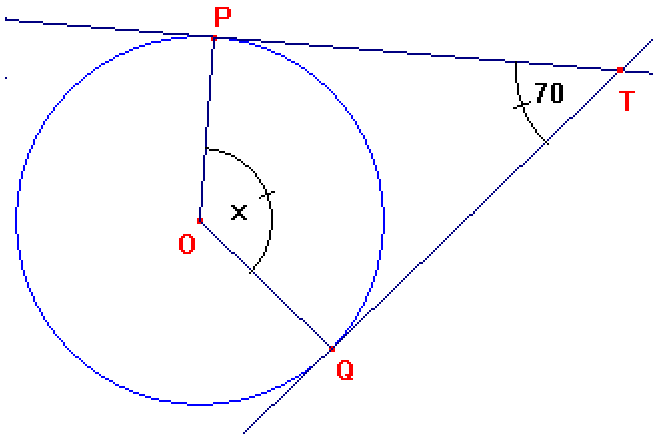
Example 2



Example 3



Example 4



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Handout

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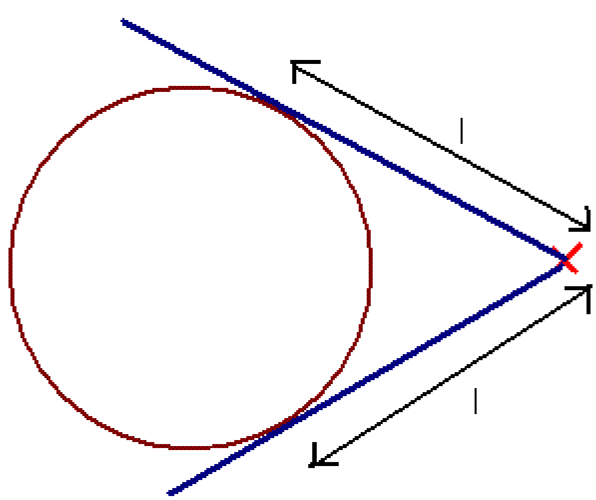
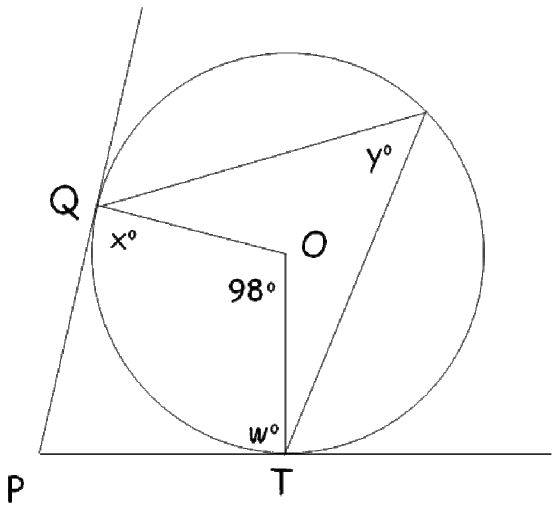
SECTION 3.7 THEOREM 6: TANGENTS FROM THE SAME EXTERNAL POINTS TO A CIRCLE ARE EQUAL IN LENGTH

If two tangents are drawn on a circle and they cross, the lengths of the two tangents (from the point where they touch the circle to the point where they cross) will be the same.

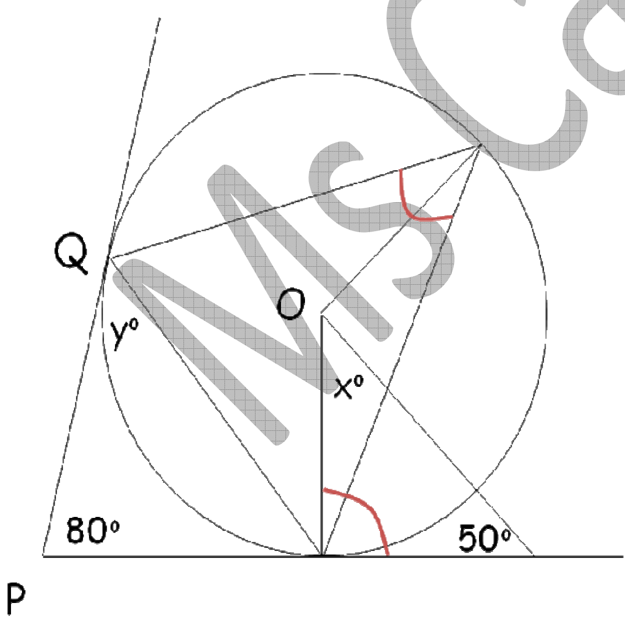


The lengths of two tangents drawn from the same external point are equal.

Example 1



Example 2



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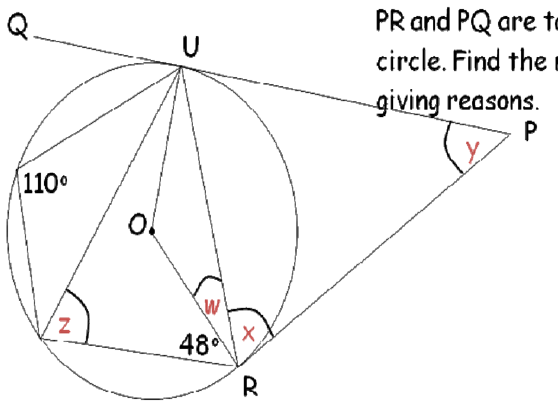
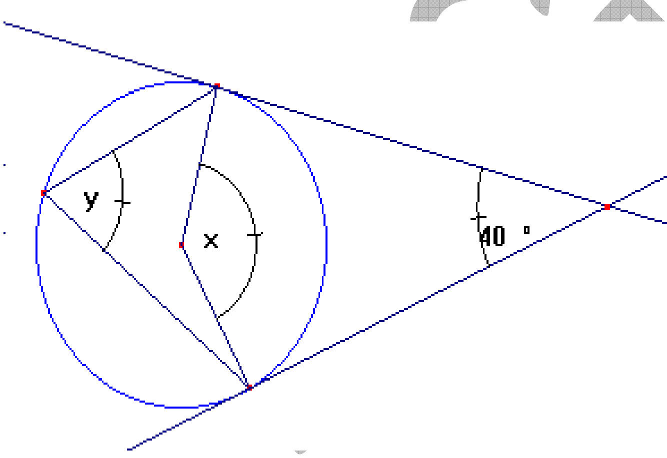
Example 3

PR and PQ are tangents to the circle. Find the missing angles giving reasons.



Example 4



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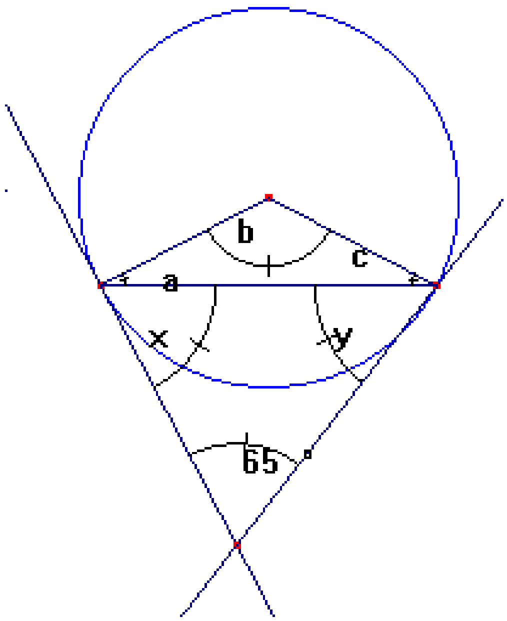
Example 5



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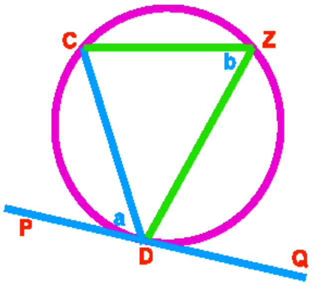
Handout

Section 3.8 Theorem 7: Alternate Segment Theorem

The angle between the tangent and chord at the point of contact is equal to the angle in the alternate segment.

This is the circle property that is the most difficult to spot. Look out for a triangle with one of its vertices (corners) resting on the point of contact of the tangent.

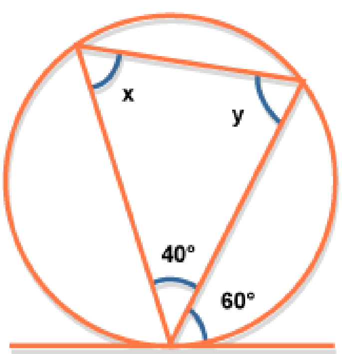


The angle between a tangent and chord is equal to the angle made by that chord in the alternate segment.

In this diagram we can use the rule to see that the yellow angles are equal, and the blue angles are equal.

Example 1

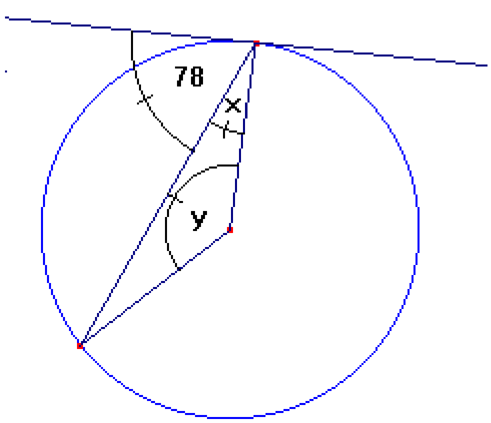
Find the angles marked with letters.



∠x = 60° [Alternate Segment Theorem]

∠y = 180° – 60° – 40° = 80° [Angles in a triangle]

Example 2



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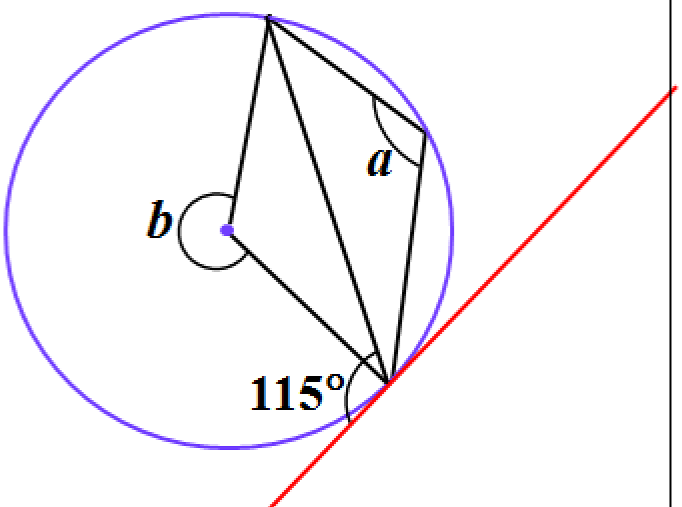
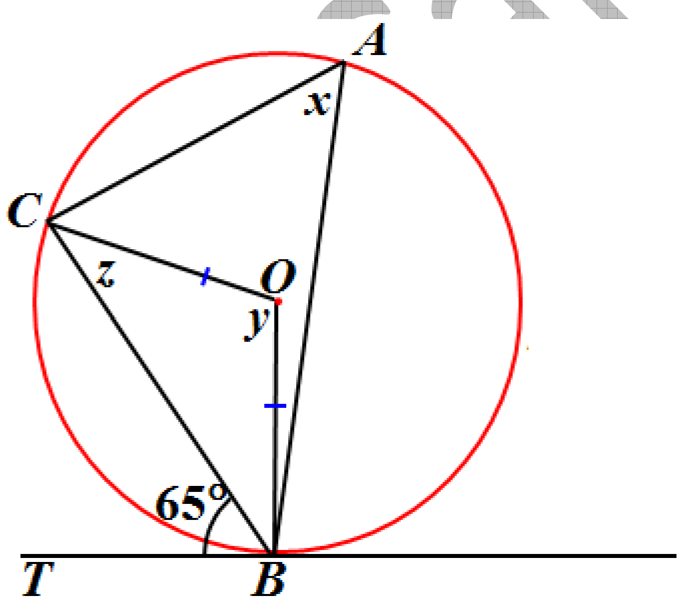
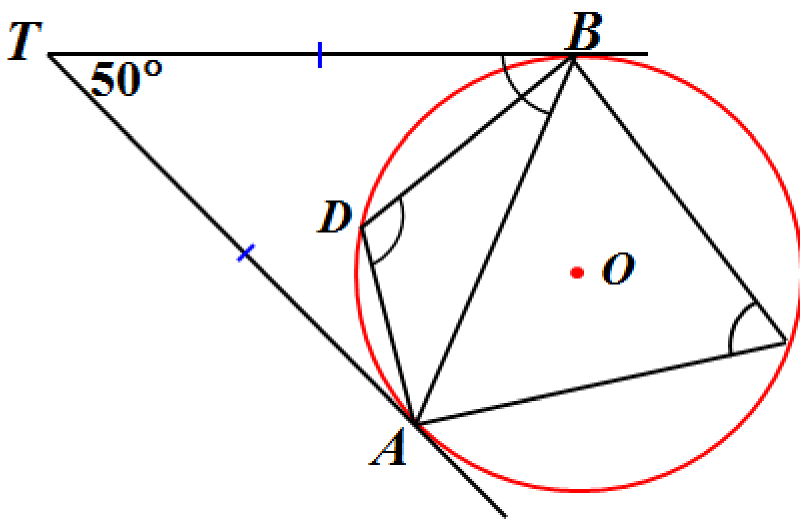
Example 3



Example 4



Example 5



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