Chapter 1 – Indices & Standard Form

Section 1.1 – Simplifying

- Only **like** (*same letters go together; same powers and same letter go together*) terms can be grouped together.

  Example: $a^2 + 3ab + 4a^2 – 5ab + 10$

  $= a^2 + 4a^2 + 3ab – 5ab + 10$

  $= 5a^2 – 2ab + 10$

- Multiplication signs are usually missed out in a simplified expression.

  Example: $2q^2 \times 3q$

  $= 2 \times q^2 \times 3 \times q$

  $= 2 \times 3 \times q^2 \times q$

  $= 6q^3$

Consolidation

1) $2a + 5b + 3a – 4b$

2) $p^2 + 4p – 6p + 2p^2 + 12$

3) $4xy – 5y – xy + y$

4) $2a \times 5b$

5) $3p^2 \times p \times 2p^3$

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Section 1.2 – Index Laws

We have 5 main index laws.

The Index Laws

$a^m \times a^n = a^{m+n}$

$\frac{a^m}{a^n} = a^{m-n}$

$(a^m)^n = a^{mn}$

$a^0 = 1$

$a^{-n} = \frac{1}{a^n}$

Consolidation: Simplify the following:-

1) $a^4 \times a^3 =$ ____________________

2) $a^6 \div a^2 =$ ____________________

3) $(a^3)^{-2} =$ ____________________

4) $-2a^4 \times 5a^{-7} =$ ____________________

5) $(2a^2)^3 =$ ____________________

6) $18a^{-2} \div 3a^{-1} =$ ______________________

7) $5a^{2}b^{4} \times 2ab^{-3} =$ ______________________

8) $24a^{-3}b^{4} \div 3a^{2}b^{-3} =$ ______________________

9) $\dfrac{3abc \times 4a^{3}b^{2}c \times 6c^{2}}{9a^{2}bc} =$ ______________________

Support Exercise Pg 73 Exercise 5C No 1 – 8

Section 1.3 Zero and Negative Powers

When we have negative powers, in order to give the value we have to always work out the reciprocal.

Work out the value of the following:-

Example: 3^{-2}

$= \dfrac{1}{3^{2}}$ *[Write the reciprocal]*

$= \dfrac{1}{9}$ *[Work out the index in the denominator]*

Example: $\left(\dfrac{2}{3}\right)^{-3}$

$\left(\dfrac{3}{2}\right)^{3}$ *[Write the reciprocal]*

$\dfrac{3^{3}}{2^{3}}$ *[Work the index in both the numerator and denominator]*

$\dfrac{27}{8} = 3\dfrac{3}{8}$ *[Work out the index]*

Example: $\frac{2^2 \times 2^5}{2^8 \times 2^3}$ *[Work the multiplication in the numerator and denominator]*

$= \frac{2^7}{2^{11}}$ *[Work the division]*

$= 2^{-4}$ *[Work the negative power]*

$= \frac{1}{2^4}$

$= \frac{1}{16}$

Consolidation: Simplify the following:-

1) 3^{-1} ______________________

2) $\left(\frac{3}{5}\right)^{-2}$ ______________________

3) $\frac{2^4 \times 2^3}{2^8 \times 2^{-1}}$ ______________________

Find the value of n for each of the following:-

1) $2^n = \frac{2^3}{2^6}$ ______________________

2) $4^2 \times 4^n = \frac{4^{11}}{4^5}$ ______________________

3) $\frac{3^n}{3^2} = \frac{3^{15}}{3^5}$ ______________________

Support Exercise Pg 426 Exercise 26A No 1 – 10

Section 1.4 Standard Form

Standard form is a way of writing down very large or very small numbers easily.

$10^3 = 1000$, so $4 \times 10^3 = 4000$. So 4000 can be written as 4×10^3 . This idea can be used to write even larger numbers down easily in standard form.

Small numbers can also be written in standard form. However, instead of the index being **positive** (in the above example, the index was 3), it will be **negative**.

The rules when writing a number in standard form is that first you write down a number between 1 and 10, then you write × 10(to the power of a number).

Example: Write 81 900 000 000 000 in standard form:

81 900 000 000 000 = 8.19×10^{13}

It's 10^{13} because the decimal point has been moved 13 places to the left to get the number to be 8.19

Example: Write 0.000 001 2 in standard form:

0.000 001 2 = 1.2×10^{-6}

It's 10^{-6} because the decimal point has been moved 6 places to the right to get the number to be 1.2

Use of Calculator

On a calculator, you usually enter a number in standard form as follows:
Type in the first number (the one between 1 and 10).

Press **EXP** .

Type in the power to which the 10 is risen.

Consolidation: Write the following as ordinary numbers:-

1) 4.2×10^{4} ____________________

2) 3.544×10^{5} ____________________

3) 2×10^{3} ____________________

4) 1.2×10^{-1} ____________________

5) 7.5×10^{-3} ____________________

6) 3×10^{0} ____________________

Consolidation: Write the following numbers in standard form:-

1) 6 000 ____________________

2) 5 ____________________

3) 0.4 ____________________

4) 0.000 259 ____________________

5) 0.001 97 ____________________

6) 375 500 ____________________

Example: Multiplication of standard form

$(\mathbf{7} \times 10^{3}) \times (\mathbf{2.3} \times 10^{-5})$

$= 16.1 \times 10^{3} \times 10^{-5}$ *[Multiply the numbers in bold together and copy the rest]*

$= 16.1 \times 10^{3+(-5)}$ *[Add the powers of the 10]*

$= 16.1 \times 10^{-2}$ *[Check whether the result is in Standard Form]*

$= 1.61 \times 10^{1} \times 10^{-2}$ *[If not write the number in Standard Form]*

$= 1.61 \times 10^{1+(-2)}$ *[Add the powers of the 10]*

$= 1.61 \times 10^{-1}$

Example: Division of standard form

$(3 \times 10^4) \div (2 \times 10^{-8})$

$$=\frac{3 \times 10^4}{2 \times 10^{-8}} = \frac{3}{2} \times \frac{10^4}{10^{-8}} = 1.5 \times 10^{4-(-8)} = 1.5 \times 10^{12}$$

Consolidation: Evaluate the following, giving your answers in standard form:-

1) $(6 \times 10^9) \times (5 \times 10^3)$ ______________________________
2) $(4 \times 10^8) \div (2 \times 10^3)$ ______________________________
3) $(3.2 \times 10^{10}) \times (6.5 \times 10^6)$ ______________________________
4) $(2.46 \times 10^{10}) \div (2.5 \times 10^3)$ ______________________________

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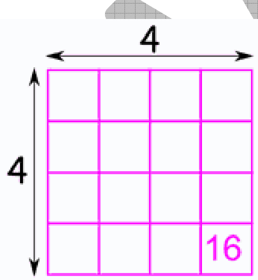
Section 1.5 Fractional Indices

What is square root?



The **square root** of a number is that special value that, when multiplied by itself, gives the number.

Example: 4 × 4 = 16, so the square root of 16 is 4.

The symbol is √

Example: √36 = 6 (because 6 x 6 = 36)

What is cube root?

The **cube root** of a number is that special value that, when used in a multiplication **three times** gives that number.

Example: 3 × 3 × 3 = 27, so the cube root of 27 is 3.

Proof of fractional indices

Using $x^m \times x^n = x^{m+n}$

$x^{½} \times x^{½} = x^{½ + ½} = x^1 = x$

This means that $x^{½}$ **multiplied** by itself gives x.

So

$x^{½}$ is the same as the **square root** of x

This means

$x^{½} = \sqrt{x}$

Similarly

$x^{⅓} \times x^{⅓} \times x^{⅓} = x$

So

$x^{⅓}$ is the same as the **cube root** of x

This means

$x^{⅓} = \sqrt[3]{x}$

In general

$x^{1/n} = \sqrt[n]{x}$ where $\sqrt[n]{x}$ means the nth root of x.

Example 1

$4^{½} = \sqrt{4} = 2$

$125^{⅓} = \sqrt[3]{125} = 5$

$27^{⅓} = \sqrt[3]{27} = 3$

What about more complicated fraction powers?

What about a fractional exponent like $4^{3/2}$? That is really saying to do a **cube** (3) and a **square root** (1/2), in any order.

Let me explain.

A fraction (like **m/n**) can be broken into two parts:

- a whole number part (**m**) , and
- a fraction (**1/n**) part

So, because **m/n = m × (1/n)** we can do this:

$$x^{\frac{m}{n}} = x^{(m\times\frac{1}{n})} = (x^m)^{\frac{1}{n}} = \sqrt[n]{x^m}$$

The order does not matter, so it also works for **m/n = (1/n) × m**:

$$x^{\frac{m}{n}} = x^{(\frac{1}{n}\times m)} = (x^{\frac{1}{n}})^m = (\sqrt[n]{x})^m$$

And we get this:

A fractional exponent of the form $\frac{m}{n}$ means:

- Do the mth power
- Work the nth root

Example 2

What is $4^{3/2}$?

$4^{3/2} = 4^{3\times(1/2)} = \sqrt{(4^3)} = \sqrt{(4\times4\times4)} = \sqrt{(64)} = \mathbf{8}$

or

$4^{3/2} = 4^{(1/2)\times3} = (\sqrt{4})^3 = (2)^3 = \mathbf{8}$

Either way gets the same result.

Example 3

What is $27^{4/3}$?

$27^{4/3} = 27^{4\times(1/3)} = \sqrt[3]{(27^4)} = \sqrt[3]{(531441)} = \mathbf{81}$

or

$27^{4/3} = 27^{(1/3)\times4} = (\sqrt[3]{27})^4 = (3)^4 = \mathbf{81}$

It was certainly easier the 2nd way!

When a fraction is raised to a power, example $\left(\frac{2}{5}\right)^3 = \frac{2}{5}\times\frac{2}{5}\times\frac{2}{5} = \frac{2^3}{5^3}$ then in general $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

Example 4

Work out

$$\frac{8}{27}^{-\frac{1}{3}} = \frac{1}{\frac{8}{27}^{\frac{1}{3}}}$$

$$\frac{27^{\frac{1}{3}}}{8} = \frac{27^{\frac{1}{3}}}{8^{\frac{1}{3}}}$$

$$\frac{27^{\frac{1}{3}}}{8^{\frac{1}{3}}} = \frac{3}{2} = 1\frac{1}{2}$$

Example 5

Work out

$$16^{-\frac{3}{4}} = \frac{1}{16^{\frac{3}{4}}} = \frac{1}{(\sqrt[4]{16})^3} = \frac{1}{2^3} = \frac{1}{8}$$

Consolidation: Evaluate the following:-

1) $32^{\frac{4}{5}}$
2) $8^{\frac{2}{3}}$
3) $25^{-\frac{1}{2}}$
4) $64^{-\frac{5}{6}}$
5) $\left(\frac{4}{9}\right)^{-\frac{5}{2}}$

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Section 1.6 Using indices to solve powers

The index laws can be used to solve for x. We shall be using only the powers to solve the equations. We must only be very careful that the base is the same everywhere.

Example 1

$5^x = 5^3$

[We can see that both the base numbers are the same (5)]

Therefore we can say that:

$x = 3$

Example 2

$10^{1-x} = 10^4$

[We can see that both the base numbers are the same (10)]

Therefore we can say that:

$1 – x = 4$ *[Add x on both sides]*

$1 = 4 + x$ *[Subtract 4 on both sides]*

$1 – 4 = x$

$-3 = x$

Example 3

$3^x = 9$

[This time we do not have the same base. We have to try and get them the same]

We can say that: $9 = 3^2$

Therefore:

$3^x = 3^2$

$x = 2$

Example 4

$2^{2x-4} = {}^1/_8$

[Can ${}^1/_8$ be written as $2^?$]

Therefore

${}^1/_8 = 2^{-3}$

$2^{2x-4} = 2^{-3}$

$2x - 4 = -3$

$2x = 4 - 3$

$2x = 1$

$x = ½$

Example 5

$4^{2x+4} = 8^{2x}$

$4 = 2^2$ and $8 = 2^3$

$(2^2)^{2x+4} = (2^3)^{2x}$

$4x + 8 = 6x$

$8 = 2x$

$x = 4$

Consolidation

Solve the following:

1) $4^{3x-1} = 16$

2) $3^{4x-10} = {}^{1}/_{9}$

3) $9^{x-4} = 27^{2+x}$

Support Exercise Handout