CHAPTER 10: FUNCTIONS

Section 10.1 Function Notation

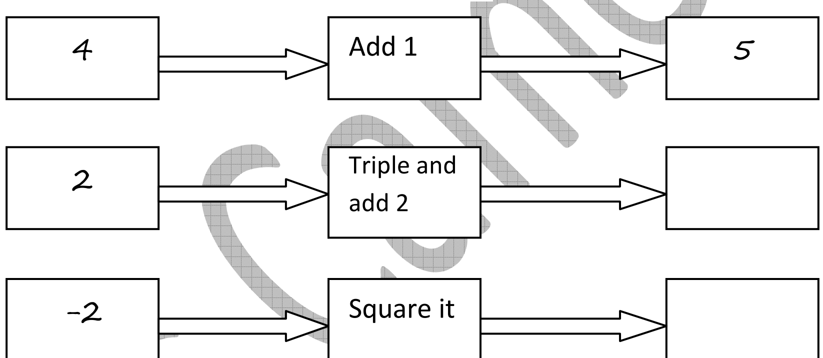
You can think of a function as being a box with a special rule… stuff goes in the box… and stuff comes out of the box.

You are familiar with equations written using x and y such as y = 3x – 4.

This equation is showing that y is a function of *x*. This means that the value of *y* depends on the value of *x* so that *y* changes when x changes.

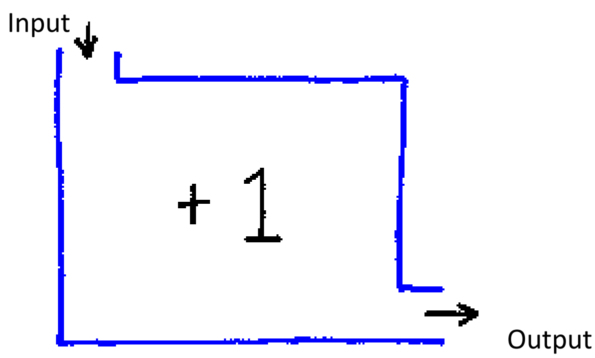
We have previously learnt these as FUNCTION MACHINES.

The Rule



Instead of boxes we need a way to talk about Functions in a mathematical way.

The Rule: Add 1



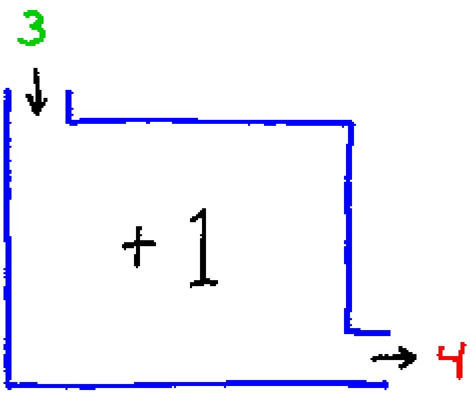
If we use x to represent the Input… the notation will be something like this:

$f(\mathbf{x}) = \mathbf{x} + 1$

Input Output

(You read this *f(x)* as, *f* **of x**)

So we can say;



It is useful to use different notation to show the above.

The function is officially written:

$f(x) = \mathbf{x + 1}$

The value of f(x) when x is 3 can be written as *f(3).*

$f(3) = 3 + 1 = 4$

$f(3) = 4$

If there are different functions in the same problem different letters can be used such as

$g(x) = 3x – 4$ or 'function of *g*'

Sometimes instead of writing $f(x) = x + 1$ we write $f{:}x \rightarrow x + 1$. These two forms mean exactly the same thing.

Consolidation

1. **$f(x)$ = 2x + 6. Find:**

   a) $f(3)$ b) $f(10)$ c) $f(\frac{1}{2})$ d) $f(-4)$

2. $g(x) = \dfrac{x^2 + 1}{2}$ **. Find:**

   a) $g(0)$ b) $g(3)$ c) $g(-2)$ d) $g(-\frac{1}{2})$

Support Exercise Edexcel Pg 583 Exercise 36A No 1 & 2

Section 10.2 Simplifying with function notations

We can use function notations in order to simplify the expression given. It is not always the case that the value to substitute in the function is a number… it can be another expressions.

Example 1

Given that *f(x)*= 2x – 1 simplify the following

a) $f(x + 4)$

$f(x + 4) = 2(x + 4) - 1$

$= 2x + 8 - 1$

$= 2x + 7$

b) $f(-5x + 7)$

c) $f(x^2 + 3x - 2)$

Example 2

Let *g(x)* = $x^2 + 4$

a) $g(x) - 1 = x^2 + 4 - 1$

$= x^2 + 3$

b) g(- x)

c) $-g(x)$

Consolidation

1. Let $f(x) = x^2 + 2x - 3$

   a) $f(x+2)$

   b) $f(2x + y)$

   c) $f(-x)$

2. If $f(x) = 3x - 4$, write the following in its simplest form:

   a) $f(2x)$

b) $f(x-1)$

c) $f(^x/_3)$

Support Exercise Edexcel Chapter 36 Exercise 36A No 3 – 5

Section 10.3 Solving with function notations

Functions can also be used to solve the value of x.

<u>Example 1</u>

$f(x)$ = 5x – 4

Find the value of x when $f(x)$ = 6

5x – 4 = 6

5x = 6 + 4

5x = 10

x = 10 ÷ 5

x = 2

Example 2

$f(x) = \frac{1}{x}$

Find the value of x when *f(x)* = 5

$\frac{1}{x} = 5$

$1 = 5x$

$\frac{1}{5} = x$

Consolidation

1. ***f(x)* = 3 – x**

   Find the value of x when *f(x)* = -4

2. ***f(x)* = 2x + 1**

   Find the value of x when *f(x)* = -9

3. ***f(x)* = 3x + 2**

   Find the value of x when *f(x)* = 11

4. ***f(x)*** **= 2x – 5**

Find the value of x when *f(x)* = 9

Support Exercise Worksheet

Section 10.4 Using two Functions

Example 1

f(x) **= 5x – 1 and *g(x)* = x + 7. Solve the equation *f(x)* = *g(x)*.**

Since *f(x)* = 5x – 1

g(x) = x + 7

If *f(x)* = *g(x)*

5x – 1 = x + 7

5x – x = 7 + 1

4x = 8

x = 2

Example 2

$f(x) = 2x - 1$ and $g(x) = 5x + 3$. Solve the equation $f(x) = g(2)$

Since $f(x) = 2x - 1$

$g(x) = 5x + 3$

$g(2) = 5(2) + 3$

$= 10 + 3$

$= 13$

If $f(x) = g(2)$

$2x - 1 = 13$

$2x = 13 + 1$

$2x = 14$

$\mathbf{x = 7}$

Consolidation

1. If $f(x) = 2x + 7$ and $g(x) = 3x - 2$, solve the equation $f(x) = g(4)$.

2. If $f(x) = 3x - 1$ and $g(x) = 2x + 1$, solve $2f(x) = 4g(x)$

3. If $f(x) = x - 1$ solve the equation $2f(x) = 3f(x)$

4. If $f(x) = x + 3$, $g(x) = x + 1$ and $h(x) = x + 2$, solve $5f(x) + 3h(x) = 2g(x)$

5. If $f(x) = 3x + 4$ and $g(x) = 4x + 7$, solve $5f(x) = 3g(x)$

Support Exercise Handout

Section 10.5 The Inverse Notation

Suppose ***f(x)* = 2x + 6**

Then *f(1)* = 8, *f(3)* = 12 and *f(-4)* = -2

The inverse of f is the function which has the opposite effect and 'undoes' f. We write the inverse of *f* as f^{-1}.

Since f above means 'multiply by 2 and then add 6', the inverse will be 'subtract 6 and then divide by 2':

$$f^{-1}(x) = \frac{x-6}{2}$$

So $f^{-1}(8) = 1$, $f^{-1}(12) = 3$ and $f^{-1}(-2) = -4$

You can find the inverse in the following way:

Step 1: Write y = *f(x)*

y = 2x + 6

Step 2: Rearrange to make x subject of the formula

y – 6 = 2x

$$\frac{y-6}{2} = x$$

$$x = \frac{y-6}{2}$$

Step 3: Replace y by x in the result.

$$f^{-1}(x) = \frac{x-6}{2}$$

Example 1

Find $f^{-1}(x)$ for the following functions

a) $f(x) = x + 7$ ***b) $f(x) = 8x$*** ***c) $f(x) = {}^{x}/_{5}$***

a) $y = x + 7$

$y - 7 = x$

$x = y - 7$

$f^{-1}(x) = x - 7$

b) $y = 8x$

${}^{y}/_{8} = x$

$x = {}^{y}/_{8}$

$f^{-1}(x) = {}^{x}/_{8}$

c) $y = {}^{x}/_{5}$

$y \times 5 = x$

$x = 5y$

$f^{-1}(x) = 5x$

Consolidation

1. Find $f^{-1}(x)$ for the following functions:

a) $f(x) = \frac{x}{3} - 2$

b) $f(x) = 4(x - 5)$

c) $f(x) = \dfrac{x+4}{5}$

d) $f(x) = \dfrac{3x-6}{2}$

e) $f(x) = 3\left(\dfrac{x}{2} + 4\right)$

f) $f(x) = 4x^3$

Support Exercise Handout